ATME COLLEGE OF ENGINEERING

13th KM Stone, Bannur Road, Mysore - 570 028

DEPARTMENT OF ELECTRICAL & ELECTRONICS ENGINEERING

NOTES

Course:Power System Analysis-1

Code:BEE601

SEMESTER: VI

INSTITUTIONAL VISION AND MISSION

VISION:

- Development of academically excellent, culturally vibrant, socially responsible and globally competent human resources.

MISSION:

- To keep pace with advancements in knowledge and make the students competitive and capable at the global level.
- To create an environment for the students to acquire the right physical, intellectual, emotional and moral foundations and shine as torchbearers of tomorrow's society.
- To strive to attain ever-higher benchmarks of educational excellence.

Department Vision and Mission

Vision:

To create Electrical and Electronics Engineers who excel to be technically competent and fulfill the cultural and social aspirations of the society.

Mission:

- To provide knowledge to students that builds a strong foundation in the basic principles of electrical engineering, problem solving abilities, analytical skills, soft skills and communication skills for their overall development.
- To offer outcome based technical education.
- To encourage faculty in training & development and to offer consultancy through research & industry interaction.

Program Educational Objectives (PEOs)

PEO1: To produce competent and ethical Electrical and Electronics Engineers who will exhibit the necessary technical and managerial skills to perform their duties in society.

PEO2: To make students continuously acquire and enhance their technical and socio-economic skills.

PEO3: To aspire students on R&D activities leading to offering solutions and excel in various career paths.

PEO4: To produce quality engineers who have the capability to work in teams and contribute to real time projects.

Program Outcomes (POs)

Engineering Graduates will be able to:

PO1: Engineering Knowledge: Apply the knowledge of mathematics, science, engineering fundamentals and an engineering specialization to the solution of complex engineering problems.

PO2: Problem Analysis: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.

PO3: Design / Development of solutions: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

PO4: Conduct investigations of complex problems: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.

PO5: Modern tool usage: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.

PO6: The engineer and society: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

PO7: Environment and sustainability: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.

PO8: Ethics: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

PO9: Individual and team work: Function effectively as an individual and as a member or leader in diverse teams, and in multidisciplinary settings.

PO10: Communication: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

PO11: Project management and finance: Demonstrate knowledge and understanding of the engineering management principles and apply these to one’s own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

PO12: Life-long learning: Recognize the need for and have the preparation and ability to engage in independent and lifelong learning in the broadest context of technological change.

Program Specific Outcomes (PSOs)

The students will develop an ability to produce the following engineering traits:

- **PSO1: Apply the concepts of Electrical & Electronics Engineering to evaluate the performance of power systems and also to control industrial drives using power electronics.**

- **PSO2: Demonstrate the concepts of process control for Industrial Automation, design models for environmental and social concerns and also exhibit continuous self- learning.**

MODULE-1

Representation of Power System Components

Course Objectives

1. To introduce the per unit system and explain its advantages and computation.
2. To explain the concept of one line diagram and its implementation in problems.

1.1 Introduction

1. A complete diagram of a power system representing all the three phases becomes too complicated for a system of practical size, so much so that it may no longer convey the information it is intended to convey.
2. It is much more practical to represent a power system by means of simple symbols for each component resulting in what is called *a* ***one-line diagram*.**
3. Per unit system leads to great simplification of three-phase networks involving transformers.
4. An impedance diagram drawn on a per unit basis does not require ideal transformers to be included in it.
5. An important element of a power system is the synchronous machine, which greatly influences the system behaviour during both steady state and transient conditions.
6. The synchronous machine model in steady state is presented in this module.
7. A 3∅ power system is **symmetrical** if impedance of all 3 phases are equal.
8. The 3∅ voltages/currents are said to be balanced if the three voltage/currents are equal in magnitude and have the same phase angle between them.
9. To plan the **operation, improvement and expansion of power system** thorough analysis is required
   - Hence it is necessary to model the power system network.
   - It is very difficult to completely model and analyse a large interconnected 3∅ power system.
   - But due to symmetry of system and balanced voltages a 3phase symmetrical balanced system can be reduced to a single phase system for analysis purpose

A power system mainly consists of generating stations, transmission lines and distribution lines. Generating stations and distribution systems are connected through transmission lines

A three phase power system is said to be symmetrical when the system is viewed from any phase is similar. Three phase voltages(currents) are said to be balanced if three voltages (or currents) are equal in magnitude and have the same phase angle difference e with respect to each other

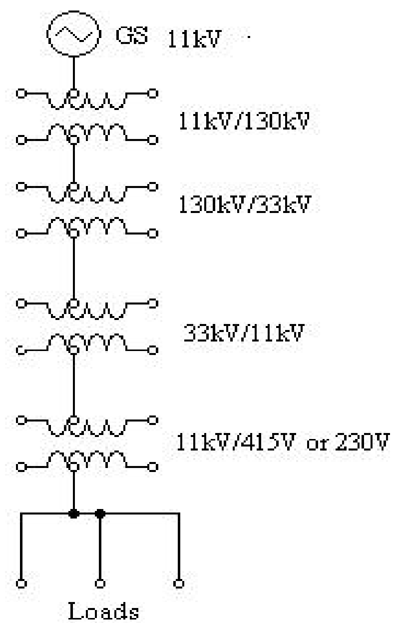


Fig 1.1 power system network

1.2 Single-phase Representation of Balanced Three Phase Networks

1 The solution of a three-phase network under balanced conditions is easily carried out by solving the **single-phase network** corresponding to the reference phase.

2 Figure 1.1 shows a simple, balanced three-phase network. The generator and load neutrals are therefore at the same potential, so that In = 0.

3 Thus the neutral impedance Zn does not affect network behaviour. For the reference phase a

$$E_a = (Z_G + Z_L)I_a \quad \text{………………….eqn.1.}$$

4 Thus the neutral impedance Zn does not affect network behaviour. For the reference phase a. The currents and voltages in the other phases have the same magnitude but are progressively shifted in phase by 120º. Eqn.1.1 corresponds to the single-phase network of **Fig. 1.2** whose solution completely determines t he solution of the three-phase network.

5 Consider now the case where a three-phase transformer forms part of a three-phase system. If the transformer is *YIY* connected as shown in Fig. 1.3(a), in the single-phase equivalent of the three-phase circuit it can be obviously represented by a single-phase transformer [as in Fig.1.3(b) with primary and secondary pertaining to phase *a* of the three-phase transformer.

6 If the transformer is *Y/Δ* connected as in Fig. 1.4(a), the delta side has to be replaced by an equivalent star connection as shown dotted so as to obtain the single-phase equivalent of Fig. 1.4(b).

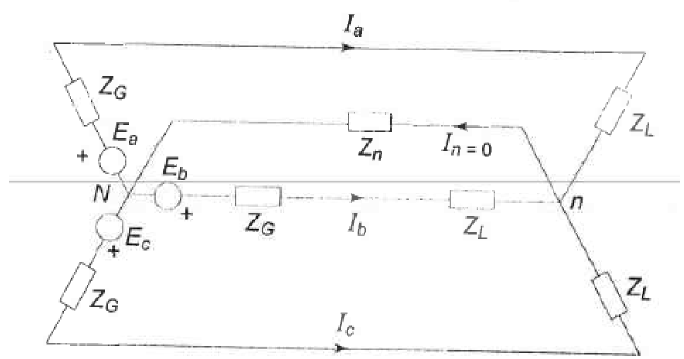


Figure 1.2: Balanced three-phase network

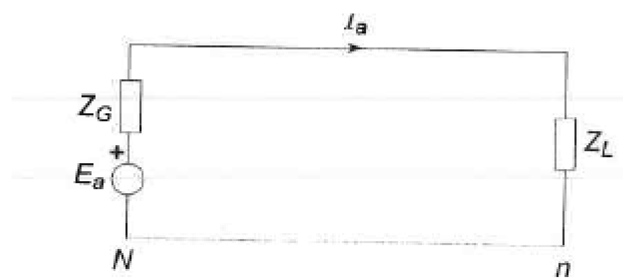
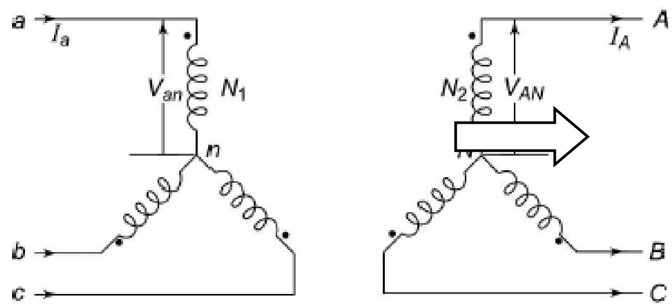
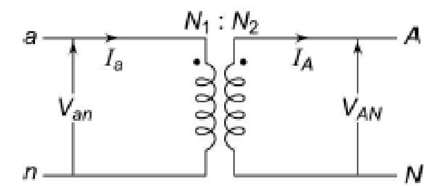


Figure 1.2: Single-phase equivalent of a balanced three-phase network of Fig. 1.1



(a) Three-phase Y/Y transformer

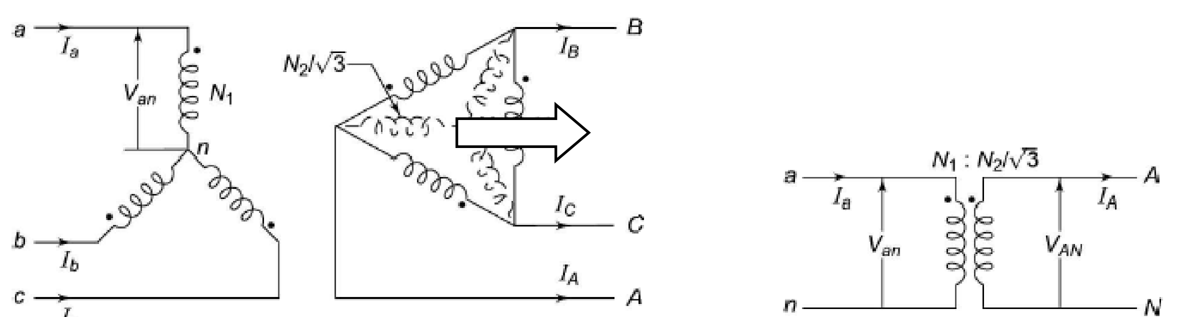


(b) Single-phase representation of three-phase Y/Y transformer

Fig. 1.3(a) Fig. 1.3(b)

10 On the delta side the voltage to neutral VAN and line current IA have a certain phase angle shift* from the star side values Van and Ia (90° for the phase labelling shown).

11 In the single- phase equivalent (VAN, IA) are respectively in phase with (Van and Ia). Since both phase voltage and line current shift through the same phase angle from star to delta side, the transformer per phase impedance and power flow are preserved in the single-phase equivalent. In most analytical studies, we are merely interested in the magnitude of voltages and currents so that the single-phase equivalent of Fig. 1.4(b) is an acceptable proposition. Wherever proper phase angles of currents and voltages are needed, correction can be easily applied after obtaining the solution through a single-phase transformer equivalent.



(a) Y/Δ transformer with equivalent star connection

(b) Single-phase equivalent of Y/Δ transformer

Fig. 1.4

1.2.1 COMPONENTS OF A POWER SYSTEM

1. Alternator

2. Power transformer

3. Transmission lines

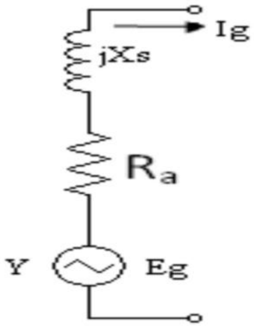
4. Substation transformer

5. Distribution transformer

6. Loads

1.3 Steady State Model of Power System Components:

a) Modelling of Generator and Synchronous Motor



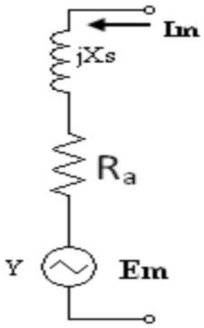


Fig 1.5: 1Φ equivalent circuit of generator **Fig 1.6:** 1Φ equivalent circuit of synchronous motor

b) Modelling of Transformer:

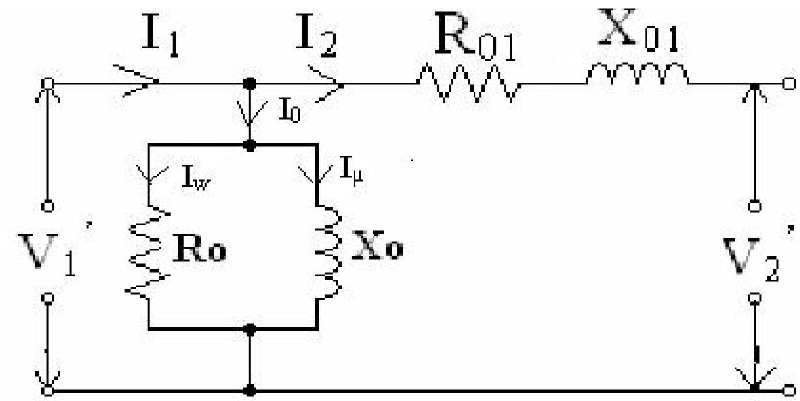


Fig 1.7: Equivalent circuit of transformer

$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}$$

$$R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2}$$ =Equivalent resistance referred to 1^o

$$X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{K^2}$$ =Equivalent reactance referred to 1^o

C) Modelling of Transmission Line:

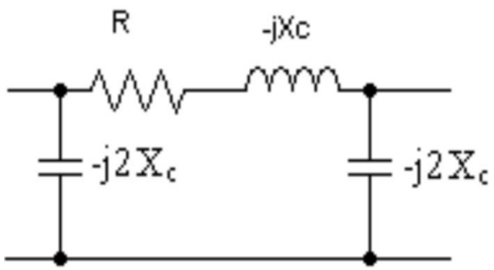


Fig 1.8: Π type

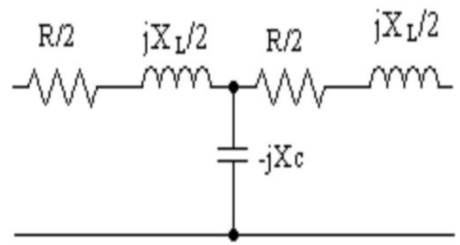


Fig 1.9: T type

d) Modelling of Induction Motor:

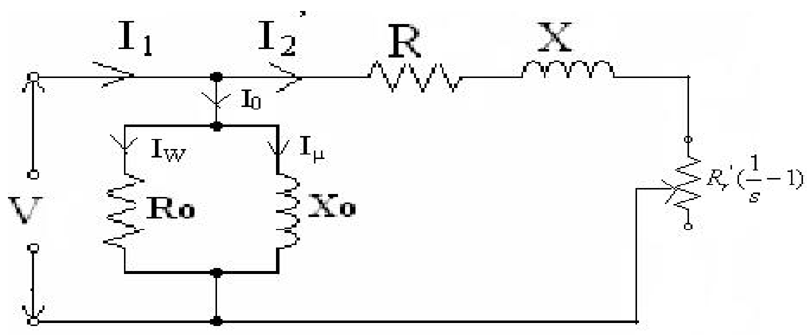


Fig 1.10: Equivalent circuit of Induction motor

$R_r'\left(\frac{1}{s}-1\right)$ =Resistance representing load

$R = R_S + R_r'$ =Equivalent resistance referred to stator

$X = X_S + X_r'$ =Equivalent reactance referred to stator

1.4 One Line Diagram

It is a diagrammatic representation of a power system in which the components are represented by their symbols.

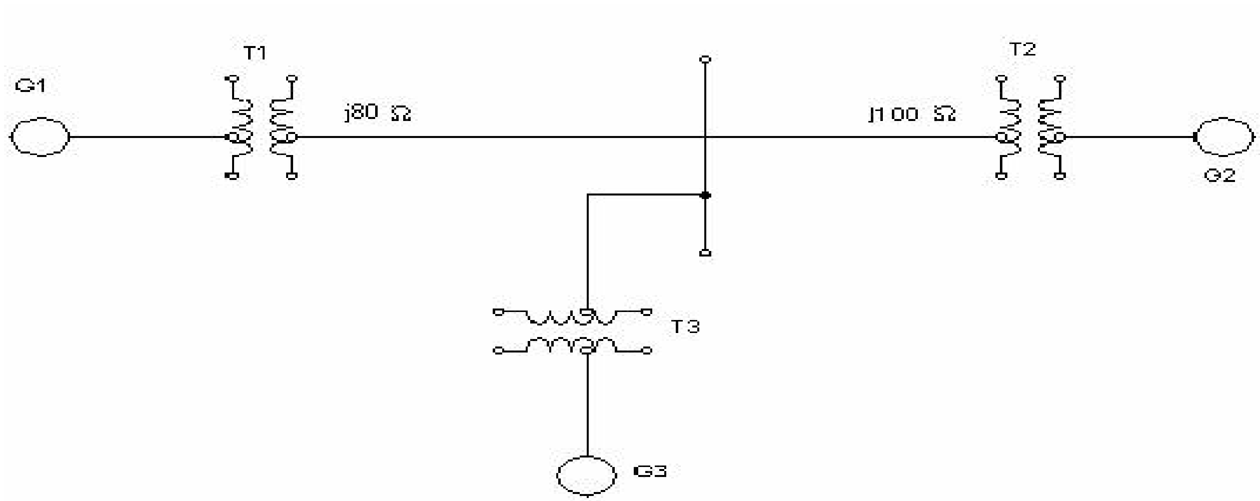
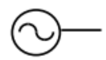
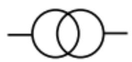
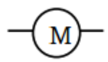


Fig 1.11: Single line diagram

Table 1.2: Power components and symbols

Power Component		Symbol	Power Component		Symbol
	=	Generator		=	Circuit breaker
	=	Transformer		=	Transmission line
	=	Motor		=	Feeder + load
	=	Busbar (substation)			

1.4.1 Impedance Diagram

This diagram obtained by replacing each component by their 1Φ equivalent circuit.

Following approximations are made to draw impedance diagram

1. The impedance b/w neutral and ground omitted.
2. Shunt branches of the transformer equivalent circuit neglected.

1.4.2 Reactance Diagram

- It is the equivalent circuit of the power system in which the various components are represented by their respective equivalent circuit.
- Reactance diagram can be obtained after omitting all resistances & capacitances of the transmission line from impedance diagram

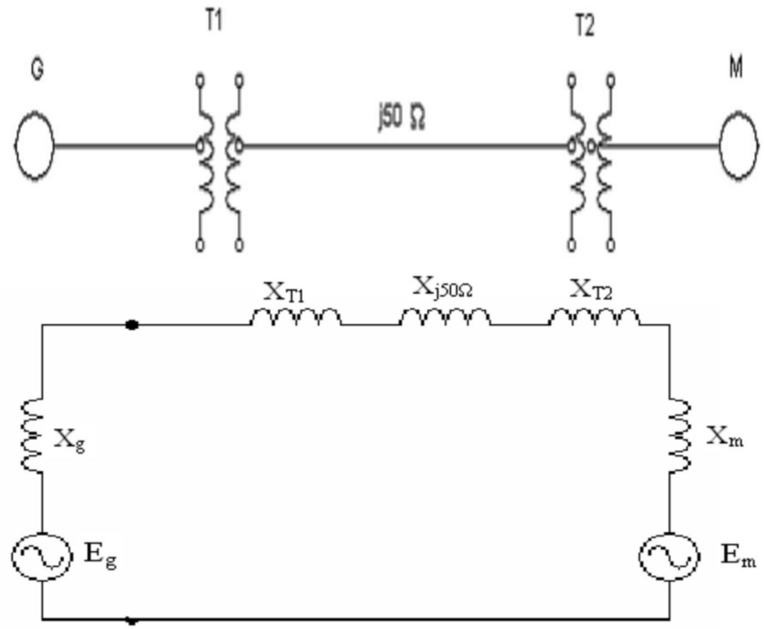


Fig 1.12: Reactance diagram for the given power system network

1.5 Per Unit System:

It is defined as the ratio of actual value of any quantity in any unit to the base value in the same unit

Per unit=Actual value/Base value

Let KVA_b=Base KVA

kV_b=Base voltage Z_b=Base impedance in **Ω**

$$Z_b = \frac{(kV_b)^2}{MVA_b} = \frac{(kV_b)^2}{\frac{KVA_b}{1000}}$$

1.5.1 CHANGING THE BASE OF PER UNIT QUANTITIES

Let z = actual impedance(**Ω**)

Z_b= base impedance (**Ω**)

$$Z_{p.u} = \frac{Z}{Z_b} = \frac{Z}{\frac{(kV_b)^2}{MVA_b}} = \frac{Z * MVA_b}{(kV_b)^2}$$

Let $KV_{b,\ old}$, $MVA_{\ b,\ old}$ represent old base values

$KV_{b,\ new}$, $MVA_{\ b,\ new}$ represent new base values

$$Z_{p.u,old} = \frac{Z * MVA_{b,old}}{(kV_{b,old})^2} \longrightarrow (1)$$

$$Z = \frac{Z_{p.u,old} * MVA_{b,old}}{(kV_{b,old})^2} \longrightarrow (2)$$

$$Z_{p.u,new} = \frac{Z * MVA_{b,new}}{(kV_{b,new})^2} \longrightarrow (3)$$

$$Z_{p.u,new} = Z_{p.u,old} * \frac{(kV_{b,old})^2}{(kV_{b,new})^2} * \frac{MVA_{b,new}}{MVA_{b,old}}$$

1.5.2 : PU Calculation of 3 Winding Transformer

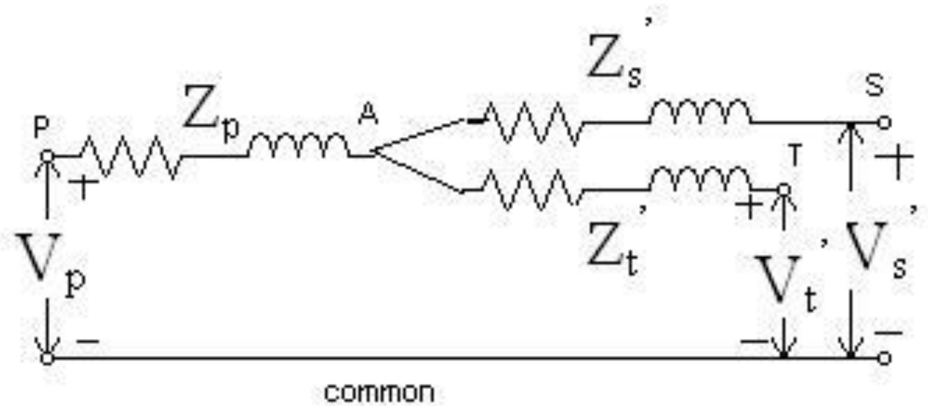


Fig 1.13: 3winding transformer

Zp=Impedance of primary winding

Zs'=Impedance of secondary winding Zt'=Impedance of tertiary winding

Short circuit test conducted to find out the above 3 impedances
Zs= ½[Zst+Zps-Zpt]
Zp= ½[Zps+Zpt- Zst]
Zt= ½[Zst+ Zpt-Zps]

Z_{ps} = Leakage impedance measured in 1^o with 2^o short circuited and tertiary open.

Z_{pt}= Leakage impedance measured in 1^o with tertiary short circuited and 2^o open.

Z_{st}' = Leakage impedance measured in 2^o with tertiary short circuited and 1^o open and referred to primary

1.5.3 ADVANTAGES OF PER UNIT CALCULATIONS

- The p.u impedance referred to either side of a 1**Φ** transformer is same
- The manufacturers provide the impedance value in p.u
- The p.u impedance referred to either side of a 3**Φ** transformer is same regardless of the 3**Φ** connections Y-Y, -Y
- P.U value always less than unity.

1.5.4 PROCEDURE TO FORM REACTANCE DIAGRAM FROM SINGLE DIAGRAM:

- Select a base power kVA_b or MVA_b
- Select a base voltage kV_b
- The voltage conversion is achieved by means of transformer kV_b on LT section= kV_b on HT section x LT voltage rating/HT voltage rating
- When specified reactance of a component is in ohms p.u reactance=actual reactance/base reactance specified reactance of a component is in p.u

$$X_{p.u,new} = X_{p.u,old} * \frac{(kV_{b,old})^2}{(kV_{b,new})^2} * \frac{MVA_{b,new}}{MVA_{b,old}}$$

Course Outcome

At the end of the module, students will be able to:

- 1.Apply the per unit system for one line diagram of power systems. [L3]

Representation Of Power System Components

- A 3ϕ power System (PS) is Symmetrical if impedance of all 3 phases are equal
- The 3ϕ voltages/currents are said to be balanced if the three voltages/currents are equal in magnitude & have same phase angle b/w them
- To plan the operation, improvement & expansion of PS, thorough analysis of it is required
- This makes us necessary to model the PS network
- It is very complicated to completely model & analyse a large interconnected 3ϕ PS
- But due to symmetry of system & balanced voltages, a 3ϕ symmetrical balanced system can be reduced to a 1ϕ system for analysis purpose

Equivalent Circuit Models Of PS Components:

- Synchronous machine, Transformers, Transmission line, Static & Dynamic loads are the various components of PS

(i) Equivalent Circuit Of Synchronous Machine:

a. Synchronous Generator:

- The 1ϕ equivalent circuit of a 3ϕ synchronous generator (balanced & symmetrical ie., same voltage & impedance in each phase) is a shown in fig 1

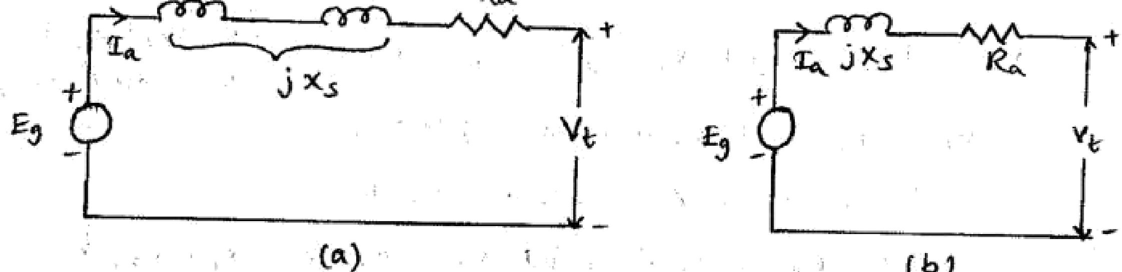


Fig 1

$V_t + I_a R_a + j I_a X_L + j I_a X_{ar} - E_g = 0$ (By applying KVL to the closed loop)

$$V_t = E_g - I_a R_a - j I_a X_L - j I_a X_{ar}$$
$$= E_g - I_a R_a - j I_a (X_L + X_{ar})$$
$$= E_g - I_a R_a - j I_a X_s \quad (\because X_s = X_{ar} + X_L)$$
$$= E_g - I_a (R_a + j X_s)$$
$$= E_g - I_a Z_s$$

Where E_g = Generated no-load voltage

$j I_a X_{ar}$ = Drop due to armature reaction reactance

$j I_a X_L$ = Drop due to armature leakage reactance

$I_a R_a$ = Drop due to armature winding resistance

X_s = Synchronous reactance of the machine

b. SYNCHRONOUS MOTOR:

- It is similar to a generator in construction
- It recieves electrical power & converts it to mechanical power
- Hence direction of current in motor is opposite to that of generator
- Its equivalent circuit is as shown in Fig 2

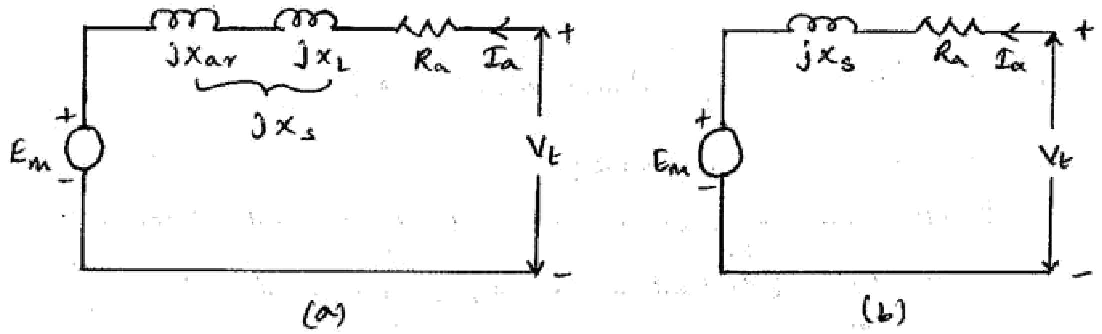


Fig 2

$V_t - I_a R_a - j I_a X_L - j I_a X_{ar} - E_m = 0$ (By applying KVL to the closed loop)

$$V_t = E_m + I_a R_a + j I_a X_L + j I_a X_{ar}$$
$$= E_m + I_a R_a + j I_a (X_L + X_{ar})$$
$$= E_m + I_a R_a + j I_a X_s \quad (\because X_s = X_L + X_{ar})$$

$$= E_m + I_a (R_a + jX_s)$$

$$= E_m + I_a Z_s$$

Where E_m = Induced voltage in motor

(ii) EQUIVALENT CIRCUIT OF A TRANSMISSION LINE (T.L):

- Short T.L length < 80km
  Medium T.L 80km < length < 250 km
  Long T.L length > 250km
- T.L is represented by nominal π-circuit or nominal T-circuit

a. NOMINAL T-CIRCUIT:

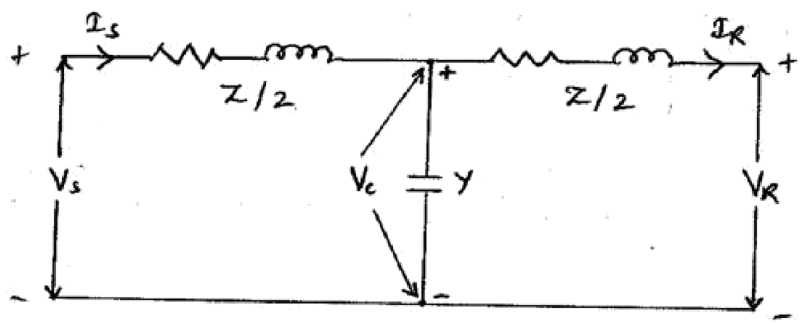


Fig 3

$$V_c = V_R + \left(\frac{Z}{2}\right) I_R \rightarrow (1)$$

$$I_s = I_R + V_c Y$$

$$= I_R + \left(V_R + \left(\frac{Z}{2}\right) I_R\right) Y \quad [\text{using } (1)]$$

$$= I_R + V_R Y + \frac{Z}{2} I_R Y$$

$$= \left(1 + \frac{ZY}{2}\right) I_R + V_R Y$$

$$I_s = Y V_R + \left(1 + \frac{ZY}{2}\right) I_R \rightarrow (2)$$

$$V_s = V_c + \frac{Z}{2} I_s$$

Using (1) & (2)

$$V_s = V_R + \frac{Z}{2} I_R + \frac{Z}{2}\left\{V_R Y + \left(1 + \frac{ZY}{2}\right) I_R\right\}$$

$$= V_R + \frac{Z}{2} I_R + \frac{Z}{2} I_R \left(1 + \frac{ZY}{2}\right) + \frac{ZY}{2} V_R$$

$$= \left(1 + \frac{ZY}{2}\right) V_R + \frac{Z}{2} I_R \left(1 + 1 + \frac{ZY}{2}\right)$$

$$= \left(1 + \frac{ZY}{2}\right) V_R + \frac{Z}{2} I_R \left(2 + \frac{ZY}{2}\right)$$

$$= \left(1 + \frac{ZY}{2}\right) V_R + \left(1 + \frac{ZY}{4}\right) Z I_R$$

W.K.T $\quad V_s = A V_R + B I_R$

$$I_s = C V_R + D I_R$$

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$= \begin{bmatrix} \left(1 + \frac{ZY}{2}\right) & Z\left(1 + \frac{ZY}{4}\right) \\ Y & \left(1 + \frac{ZY}{2}\right) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$\therefore \quad A = D = 1 + \frac{ZY}{2} \quad$ p.u.

$$B = Z\left(1 + \frac{ZY}{4}\right) \ \Omega$$

$$C = Y \quad S \text{ (Siemens)}$$

b. <u>NOMINAL</u> <u>π - CIRCUIT</u>:

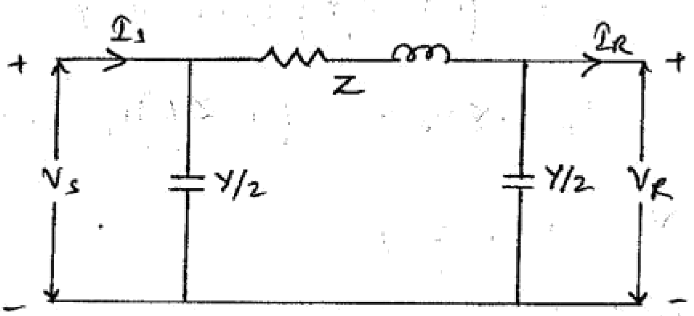


fig 4

$$I_s = I_R + V_R \cdot \frac{Y}{2} + V_s \cdot \frac{Y}{2} \rightarrow (1)$$

$$V_s = V_R + \left(I_R + V_R \cdot \frac{Y}{2}\right) Z$$

$$= V_R + Z I_R + \frac{ZY}{2} V_R$$

$$V_s = \left(1 + \frac{ZY}{2}\right) V_R + Z I_R \rightarrow (2)$$

Using (2) in (1)

$$I_s = I_R + V_R \cdot \frac{Y}{2} + \left[\left(1 + \frac{ZY}{2}\right) V_R + Z I_R\right] \frac{Y}{2}$$

$$= I_R + V_R \cdot \frac{Y}{2} + \left(1 + \frac{ZY}{2}\right) V_R \cdot \frac{Y}{2} + \frac{ZY}{2} I_R$$

$$= I_R \left(1 + \frac{ZY}{2}\right) + V_R \cdot \frac{Y}{2} \left(1 + 1 + \frac{ZY}{2}\right)$$

$$= \left(1 + \frac{ZY}{2}\right) I_R + V_R \frac{Y}{2} \left(2 + \frac{ZY}{2}\right)$$

$$= Y V_R \left(1 + \frac{ZY}{4}\right) + \left(1 + \frac{ZY}{2}\right) I_R$$

$$I_s = \left(1 + \frac{ZY}{4}\right) Y V_R + \left(1 + \frac{ZY}{2}\right) I_R$$

W.K.T

$$V_s = A V_R + B I_R$$

$$I_s = C V_R + D I_R$$

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$= \begin{bmatrix} \left(1 + \frac{ZY}{2}\right) & Z \\ Y\left(1 + \frac{ZY}{4}\right) & \left(1 + \frac{ZY}{2}\right) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\therefore \quad A = D = 1 + \frac{ZY}{2} \quad p.u.$$

$$B = Z \ \Omega$$

$$C = Y\left(1 + \frac{ZY}{4}\right) \ S \text{ (Siemen)}$$

Where

V_S & I_S = Sending end voltage & current respectively

V_R & I_R = Recieving end voltage & current respectively

$Z = zl$ = Total series impedance / phase

$(Z = zl = (R + j\omega L)l)$

$Y = yl$ = Total shunt admittance / phase

l = length of transmission line

z = Series impedance / unit length / phase

y = Shunt admittance / unit length / phase

▷ Mostly nominal π - circuit is considered in PS

(iii) <u>EQUIVALENT CIRCUIT OF TRANSFORMER (TWO WINDING)</u>:

▷ The equivalent circuit of a two winding transformer referred from primary side is as shown

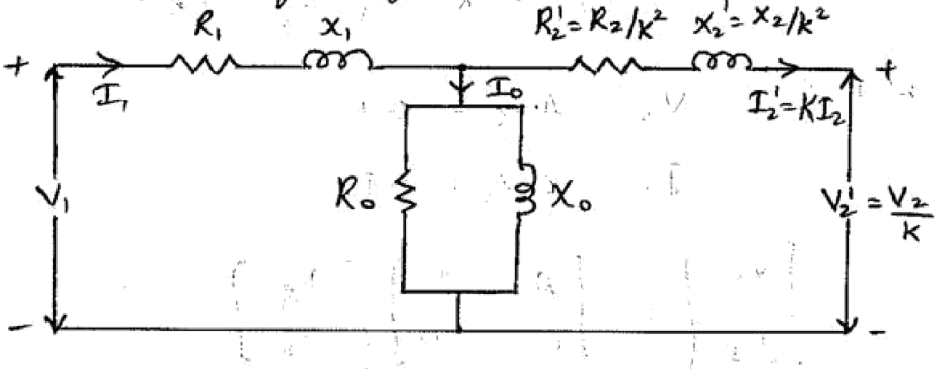


Fig 5

Where R_1, R_2 winding resistances of primary & secondary side

X_1, X_2 Leakage reactances of primary & secondary side

Voltage transformation ratio $K = \frac{V_2}{V_1} = \frac{N_2}{N_1}$

R_0 & X_0 forms exciting circuit representing magnetising current & core loss

- In a 2 winding transformer both the primary & secondary winding have same KVA rating

(iv) EQUIVALENT CIRCUIT OF THREE WINDING TRANSFORMER:

- Primary, secondary & tertiary winding designate the 3 windings
- All three windings may have different KVA rating
- Impedance of these windings are connected in star to represent 1ø equivalent circuit
- Here magnetising current is neglected
- Common point is unrelated to neutral of system
- Tertiary winding is also called stabilization winding

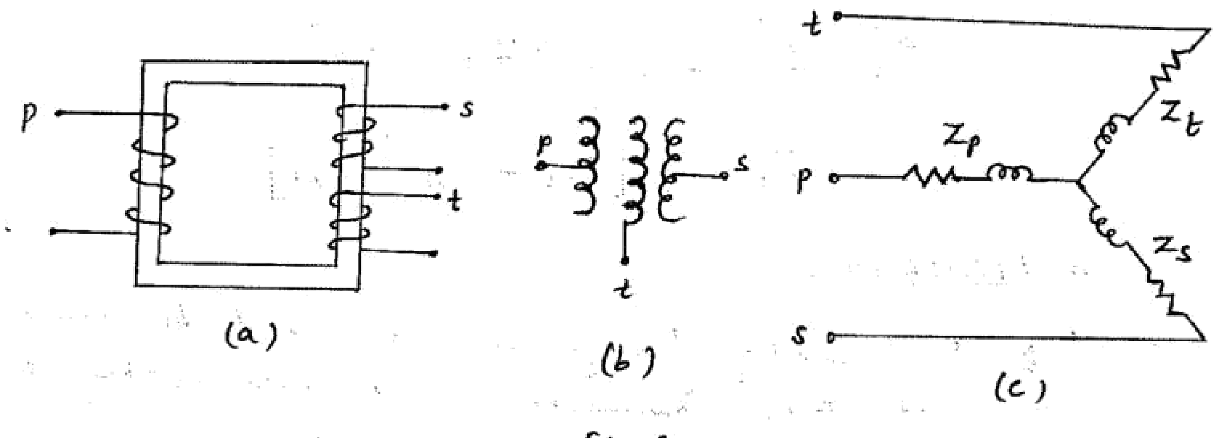


Fig 6

where

Z_p is impedance of primary winding

Z_s is impedance of secondary winding

Z_t is impedance of tertiary winding

a. PER UNIT IMPEDANCE OF THREE WINDING TRANSFORMER:

Z_{ps} = Leakage impedance measured in primary with secondary short circuited & Tertiary open

Z_{pt} = Leakage impedance measured in primary with tertiary short circuited & secondary open

Z_{st} = Leakage impedance measured in secondary with tertiary circuited & primary open

▷ Accordingly we can write the formulaes as

$$Z_{ps} = Z_p + Z_s \rightarrow (1)$$
$$Z_{pt} = Z_p + Z_t \rightarrow (2)$$
$$Z_{st} = Z_s + Z_t \rightarrow (3)$$

▷ Solving for (1) + (2) - (3) we get

$$Z_{ps} + Z_{pt} - Z_{st} = 2Z_p$$

$$\therefore Z_p = \frac{1}{2}\left[Z_{ps} + Z_{pt} - Z_{st}\right]$$

$$|||^{ly} \quad Z_s = \frac{1}{2}\left[Z_{ps} + Z_{st} - Z_{pt}\right]$$

$$Z_t = \frac{1}{2}\left[Z_{pt} + Z_{st} - Z_{ps}\right]$$

b. APPLICATION:

▷ Three winding transformers are used to interconnect three transmission lines, each working at different voltage & power levels

▷ Tertiary winding is used to supply voltage for substations auxiliary devices like compressors, feed water pumps etc, because these devices have different voltage levels than the primary & secondary windings.

- If loading is unbalanced & if we use a Y-Δ transformer then on Δ side zero sequence currents (ZSC) i.e., 3rd harmonic currents can flow, because Δ appears as a closed ckt for ZSC.
  So to provide path for ZSC in transformer tertiary winding is provided & is normally connected in Δ.
  But if we use Y-Y transformer the ZSC cannot flow leading to shifting of neutral
- Tertiary winding is also used to measure voltage of HV testing transformer
- Static capacitors or synchronous condensors are connected to tertiary winding for reactive power injection into the system
- When load is unbalanced Δ connected tertiary limits voltage imbalance
- Three winding transformers find extensive utilisation in HV labs

(V) EQUIVALENT CIRCUIT OF LOAD:

- Load drawn by consumers is the toughest parameter to assess scientifically
- Loads are composed of industrial & domestic components
- Industrial loads mainly consist of large 3ϕ induction motors with constant load & predictable duty cycle
- Domestic load mainly consists of lighting, heating & many 1ϕ devices used randomly by householders
- Nature & magnitude of loads greatly influences the economical & electrical design & operation of PS
- In representing loads for load flow & stability studies it is important to know variation of real &

reactive power with variation of voltage

- In such studies load is composite in nature with both industrial & domestic components

- Typical composition of load at bus is

Induction Motor	55 – 75%
Synchronous Motor	5 – 15%
Lighting & Heating	20 – 30%

- Loads are classifies as
  - → Static loads
  - → Dynamic loads

a. EQUIVALENT CIRCUIT OF STATIC LOADS:

- Electric furnaces, induction heaters, lamps, resistive & reactive loads etc are termed as static loads

- They are represented in equivalent circuit by any one of these
  - → Constant power representation
  - → Constant current representation
  - → Constant impedance representation

- CONSTANT POWER REPRESENTATION:
  - → Load active power (MW) & reactive power (MVAr) are considered to be constant throughout the study
  - → Used in load flow studies

- CONSTANT CURRENT REPRESENTATION:
  - → Here magnitude of load current is considered constant throughout the study

→ The current of load is calculated from specified voltage, active & reactive powers of load

→ FOR 1ϕ LOAD:

Let P = Active power
Q = Reactive power
$V = |V|\angle\delta$ be voltage of load
Its conjugate is $V^* = |V|\angle-\delta$

Complex power $S = P + jQ = VI^*$

$$(VI^*)^* = (P+jQ)^*$$

$$V^*I = P - jQ$$

$$I = \frac{P-jQ}{V^*} \rightarrow (1)$$

$$= \frac{\sqrt{P^2+Q^2}\angle-\theta}{|V|\angle-\delta}$$

Where $\theta = \tan^{-1}\left(\frac{Q}{P}\right)$ & is termed as Power factor angle

$$= \frac{\sqrt{P^2+Q^2}}{|V|}\angle\delta-\theta$$

$$I = |I|\angle\delta-\theta$$

where $|I| = \frac{\sqrt{P^2+Q^2}}{|V|}$

▷ CONSTANT IMPEDANCE REPRESENTATION:

→ Here impedance or admittance of load is considered constant throughout the study

→ Impedance of the load is calculated from the specified voltage, active & reactive powers of load

→ Quite oftenly used in stability studies

→ **For 1φ Load:**

Load Impedance $Z = \frac{V}{I}$

from ①

$$Z = \frac{V}{\frac{P-jQ}{V^*}}$$

$$= \frac{V \cdot V^*}{P-jQ}$$

$$= \frac{|V|\angle\delta \cdot |V|\angle-\delta}{P-jQ}$$

$$= \frac{|V|^2}{P-jQ}$$

Load Admittance $Y = \frac{1}{Z} = \frac{P-jQ}{|V|^2}$

▷ **For 3φ Load:**

→ **Balanced Star Connected Load:**

Let P = 3φ active power of star connected load in watt

Q = 3φ reactive power of star connected load in VAr

V, V_L = phase & line voltage of load respectively

I, I_L = phase & line current of load respectively

3φ Complex power $S = P + jQ = 3VI^*$

$$(3VI^*)^* = (P+jQ)^*$$

$$3V^*I = P - jQ$$

$$I = \frac{P-jQ}{3V^*} \rightarrow ①$$

$$= \frac{\sqrt{P^2 + Q^2} \angle -\theta}{3 |V| \angle -\delta}$$

Where $\theta = \tan^{-1}\left(\frac{Q}{P}\right)$ & is termed as power factor angle

W.K.T in Star connection

$$I = I_L \quad \& \quad |V| = \frac{|V_L|}{\sqrt{3}}$$

$$I = I_L = \frac{\sqrt{P^2 + Q^2} \angle -\theta}{3 \frac{|V_L|}{\sqrt{3}} \angle -\delta}$$

$$= \frac{\sqrt{P^2 + Q^2}}{\sqrt{3} |V_L|} \angle \delta - \theta$$

$$= |I| \angle \delta - \theta$$

$$|I| = |I_L| = \frac{\sqrt{P^2 + Q^2}}{\sqrt{3} |V_L|}$$

Load Impedance/phase $Z = \frac{V}{I}$

from ①

$$Z = \frac{V}{\frac{P - jQ}{3V^*}}$$

$$= \frac{3V \cdot V^*}{P - jQ}$$

$$= \frac{3 |V| \angle \delta \cdot |V| \angle -\delta}{P - jQ}$$

$$= \frac{3 |V|^2}{P - jQ} = \frac{3\left(|V_L| / \sqrt{3}\right)^2}{P - jQ} = \frac{|V_L|^2}{P - jQ}$$

Load Admittance/phase $Y = \frac{1}{Z} = \frac{P - jQ}{|V_L|^2}$

→ BALANCED DELTA CONNECTED LOAD:

Let $P = 3\phi$ active power of Δ connected load in Watt

$Q = 3\phi$ reactive power of Δ connected load in MVAr

V, V_L = Phase & line voltage of load respectively

I, I_L = Phase & line current of load respectively

3ϕ complex power $S = P + jQ = 3VI^*$

$$(3VI^*)^* = (P+jQ)^*$$

$$3V^* I = P - jQ$$

$$I = \frac{P-jQ}{3V^*} \rightarrow (1)$$

W.K.T in Δ, $V = V_L$ & $|I_L| = \sqrt{3}\,|I|$

$$I = \frac{\sqrt{P^2+Q^2}\,\angle -\theta}{3\,|V|\,\angle -\delta}$$

where $\theta = \tan^{-1}\left(\frac{Q}{P}\right)$ & is termed as power factor angle

$$= \frac{\sqrt{P^2+Q^2}}{3|V_L|}\,\angle \delta - \theta$$

$$I = |I|\,\angle \delta - \theta$$

where $|I| = \frac{\sqrt{P^2+Q^2}}{3|V_L|}$

$$|I_L| = \sqrt{3}\,|I|$$

$$= \sqrt{3}\cdot\frac{\sqrt{P^2+Q^2}}{3|V_L|}$$

$$= \frac{\sqrt{P^2+Q^2}}{\sqrt{3}\,|V_L|}$$

Load impedance/phase $Z = \frac{V}{I}$

from ①

$$Z = \frac{V}{\frac{P-jQ}{3V^*}}$$

$$= \frac{3\,V \cdot V^*}{P-jQ}$$

$$= \frac{3|V|\angle\delta \cdot |V|\angle{-\delta}}{P-jQ}$$

$$= \frac{3|V|^2}{P-jQ} = \frac{3\left(|V_L|\right)^2}{P-jQ} = \frac{3|V_L|^2}{P-jQ}$$

fig 7

Load Admittance/phase $Y = \frac{1}{Z} = \frac{P-jQ}{3|V_L|^2}$

▷ for any type of static load the equivalent circuit is represented with its impedance as shown in fig 7

b. <u>EQUIVALENT CIRCUIT OF DYNAMIC LOADS:</u>

▷ Synchronous Motor & induction motor forms the dynamic load in PS

▷ Induction Motor equivalent circuit referred to stator is as shown in fig 8

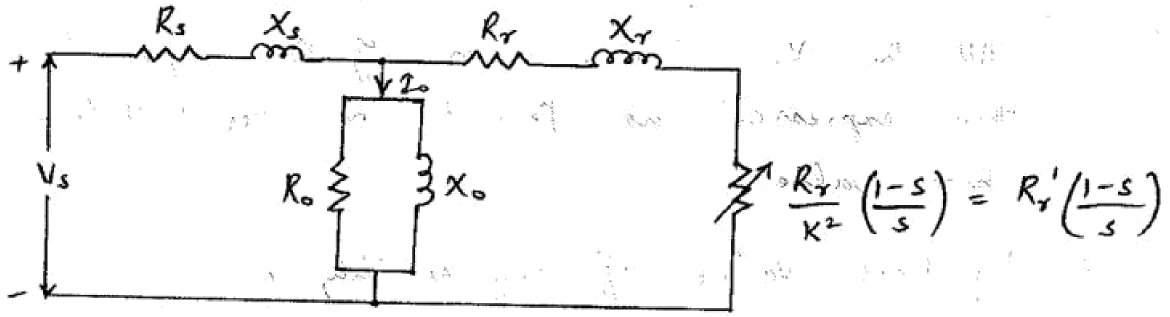


fig. 8

Where X_s, R_s = Reactance & Resistance of stator

X_r, R_r = Reactance & Resistance of rotor

R_o, X_o = forms exciting/magnetising circuit

K = Voltage transformation ratio

$R_r'\left(\frac{1-s}{s}\right)$ = Variable resistance representing load

- The equivalent circuit is similar to equivalent circuit of transformer
- Induction Motor is also called rotating transformer

PER UNIT QUANTITY:

- Per Unit (P.U.) quantity can also be termed as per phase quantity
- The PS components (alternators, motors, transformers etc) are represented with their electrical quantities (voltage (V), Power (P), Current (I), Impedance (Z))
- These electrical quantities have ratings in KV (V), KVA/MVA (S), KA (I), & Ω (Z)
- Various components/sections of PS operates at different voltage & power levels
- So to analyse PS it is necessary/convinient to express rating of electrical quantities of PS components with reference to a common value called base value.
- Therefore a base value is choosen for V, P, I & Z
- All the V, P, I & Z rating of PS Components are then expressed as Percent or Per Unit (P.U.) of base value
- Per Unit value of any quantity is defined as "Ratio of actual to base quantity expressed as a decimal"

$$\text{Per Unit Quantity} = \frac{\text{Actual quantity in any Unit}}{\text{Base quantity in same unit}}$$

Per unit quantity is dimensionless because both actual & base quantities are in same units

▷ Percent Quantity = per Unit Quantity × 100

percent quantity is not convinient for use as factor of 100 has to be carried in computation

▷ In PS base values of 4 quantities are required V, P, I & Z

▷ If base values of any 2 quantities are selected then we can determine the base values of remaining 2 quantities

CHOICE OF BASE QUANTITY:

▷ Generally two independent base values are selected for P.U. representation namely

V_{base} (or V_b) – Voltage base value in KiloVolt i.e, KV_b

S_{base} (or S_b) – Complex power base value in MegaVoltAmper or KiloVoltAmpere i.e., MVA_b / KVA_b

(i) SINGLE PHASE SYSTEM:

▷ Let $P_{b1\phi} = Q_{b1\phi} = S_{b1\phi}$

i.e, $MW_{b1\phi} = MVAr_{b1\phi} = MVA_{b1\phi}$

or $KW_{b1\phi} = KVAr_{b1\phi} = KVA_{b1\phi}$

i.e., we choose a complex power base such that it will also be the base value for real & reactive power

▷ Also let the line-to-neutral base kilovolt be KV_{bLN}

▷ Then Base Current $I_b = \dfrac{S_{b1\phi}}{V_{bLN}} = \dfrac{MVA_{b1\phi}}{KV_{bLN}}$ → (1)

$$= \frac{MVA_{b1\phi} \times 10^6}{KV_{bLN} \times 10^3} = \frac{MVA_{b1\phi} \times 1000}{KV_{bLN}}$$

$$I_b = \frac{KVA_{b1\phi}}{KV_{bLN}} \text{ A}$$

▷ Base Impedance $Z_b = R_b = X_b = \frac{V_{bLN}}{I_b}$

i.e., Impedance base is choosen such that it will also be the base value for resistance & reactance

from ①

$$Z_b = \frac{V_{bLN}}{\left(\frac{S_{b1\phi}}{V_{bLN}}\right)} = \frac{V_{bLN}^2}{S_{b1\phi}} = \frac{KV_{bLN}^2}{MVA_{b1\phi}} \rightarrow ②$$

$$= \frac{KV_{bLN}^2 \times (10^3)^2}{MVA_{b1\phi} \times 10^6} = \frac{KV_{bLN}^2 \times 1000}{MVA_{b1\phi} \times 1000}$$ (multiplying & dividing by 1000)

$$Z_b = \frac{KV_{bLN}^2 \times 1000}{KVA_{b1\phi}} \ \Omega$$

▷ Per Unit Impedance $Z_{p.u.} = \frac{Z}{Z_b}$

from ②

$$Z_{p.u.} = \frac{Z}{\left(\frac{KV_{bLN}^2}{MVA_{b1\phi}}\right)} = \frac{Z \times MVA_{b1\phi}}{KV_{bLN}^2}$$

$$= \frac{Z \times KVA_{b1\phi}}{KV_{bLN}^2 \times 1000}$$

▷ Base Admittance $Y_b = G_b = B_b = \frac{1}{Z_b}$

i.e., Admittance base is selected such that it will also be the base value for conductance & Susceptance

(ii) THREE PHASE SYSTEM:

▷ In a 3ϕ system the 1ϕ rating is $\frac{1}{3}^{rd}$ of 3ϕ rating

i.e., $$S_{b1\phi} = \frac{S_{b3\phi}}{3} \rightarrow (1)$$

▷ Let $S_{b3\phi} = P_{b3\phi} = Q_{b3\phi}$

i.e., $MVA_{b3\phi} = MW_{b3\phi} = MVAr_{b3\phi}$

or $KVA_{b3\phi} = KW_{b3\phi} = KVAr_{b3\phi}$

i.e., we choose 3ϕ base for Complex power such that it will also be the 3ϕ base value for real & reactive power

▷ Also instead of using line-to-neutral base kilovolt KV_{bLN} we consider line-to-line kilovolt KV_{bLL}

▷ Assuming Star Connection the relationship b/w V_{bLN} & V_{bLL} is

$$V_{bLN} = \frac{V_{bLL}}{\sqrt{3}} \rightarrow (2)$$

▷ Base Current $$I_b = \frac{S_{b1\phi}}{V_{bLN}} \rightarrow (3)$$

from (1) & (2)

$$I_b = \frac{S_{b3\phi}/3}{V_{bLL}/\sqrt{3}} = \frac{S_{b3\phi}}{3} \times \frac{\sqrt{3}}{V_{bLL}}$$

$$= \frac{S_{b3\phi}}{\sqrt{3}\cdot\sqrt{3}} \times \frac{\sqrt{3}}{V_{bLL}}$$

$$= \frac{S_{b3\phi}}{\sqrt{3}\ V_{bLL}} = \frac{MVA_{b3\phi}}{\sqrt{3}\ KV_{bLL}} = \frac{MVA_{b3\phi} \times 10^6}{\sqrt{3} \times KV_{bLL} \times 10^3}$$

$$= \frac{MVA_{b3\phi} \times 1000}{\sqrt{3}\ KV_{bLL}}$$

$$I_b = \frac{KVA_{b3\phi}}{\sqrt{3}\ KV_{bLL}} \quad A$$

▷ Base Impedance $Z_b = R_b = X_b = \dfrac{V_{bLN}}{I_b}$

from ③

$$Z_b = \frac{V_{bLN}}{\left(\frac{S_{b1\phi}}{V_{bLN}}\right)} = \frac{V_{bLN}^2}{S_{b1\phi}}$$

from ① & ②

$$Z_b = \frac{\left(V_{bLL}/\sqrt{3}\right)^2}{S_{b3\phi}/3}$$

$$= \frac{V_{bLL}^2}{3} \times \frac{3}{S_{b3\phi}}$$

$$= \frac{V_{bLL}^2}{S_{b3\phi}} = \frac{KV_{bLL}^2}{MVA_{b3\phi}} \rightarrow ④$$

$$= \frac{KV_{bLL}^2 \times (10^3)^2}{MVA_{b3\phi} \times 10^6} = \frac{KV_{bLL}^2 \times 1000}{MVA_{b3\phi} \times 1000}$$ (multiplying & dividing by 1000)

$$Z_b = \frac{KV_{bLL}^2 \times 1000}{KVA_{b3\phi}} \quad \Omega$$

▷ Per Unit Impedance $Z_{p.u.} = \frac{Z}{Z_b}$

from (4)

$$Z_{p.u.} = \frac{Z}{\left(\frac{KV_{bLL}^2}{MVA_{b3\phi}}\right)} = \frac{Z \times MVA_{b3\phi}}{KV_{bLL}^2}$$

$$Z_{p.u.} = \frac{Z \times KVA_{b3\phi}}{KV_{bLL}^2 \times 1000}$$

▷ Base Admittance $Y_b = G_b = B_b = \frac{1}{Z_b}$

CHANGE OF BASE:

▷ In PS there are equipments with different ratings, so it is necessary to convert impedance value from one rating to another

▷ This is because equipments manufacturers generally specify the value of impedance of any equipment in per unit or percent quantity based on name plate rating of that equipment

▷ We may choose a different base value for voltage or power for the whole PS (where all these equipments with their own per unit impedances are interconnect -ed)

▷ Therefore we need to convert impedance from one base value to another

▷ Let $Z_{b,old}$, $S_{b,old}$ (i.e., $MVA_{b,old}$) & $V_{b,old}$ (i.e., $KV_{b,old}$) represent old base value

$Z_{b,new}$, $S_{b,new}$ (i.e, $MVA_{b,new}$) & $V_{b,new}$ (i.e, $KV_{b,new}$) represent new base value

Then

$$Z_{b,old} = \frac{V_{b,old}^2}{S_{b,old}} = \frac{KV_{b,old}^2}{MVA_{b,old}} \rightarrow (1)$$

$$Z_{b,new} = \frac{V_{b,new}^2}{S_{b,new}} = \frac{KV_{b,new}^2}{MVA_{b,new}} \rightarrow (2)$$

$$Z_{p.u.,old} = \frac{Z_{actual}}{Z_{b,old}}$$

from (1)

$$Z_{p.u.,old} \times \frac{KV_{b,old}^2}{MVA_{b,old}} = Z_{actual}$$

$$Z_{actual} = Z_{p.u.,old} \times \frac{KV_{b,old}^2}{MVA_{b,old}} = Z_{p.u.,old} \times \frac{V_{b,old}^2}{S_{b,old}} \rightarrow (3)$$

$$Z_{p.u.,new} = \frac{Z_{actual}}{Z_{b,new}}$$

from (2) & (3)

$$= \frac{Z_{p.u.,old} \times \frac{V_{b,old}^2}{S_{b,old}}}{\frac{V_{b,new}^2}{S_{b,new}}}$$

$$= Z_{p.u.,old} \times \frac{V_{b,old}^2}{S_{b,old}} \times \frac{S_{b,new}}{V_{b,new}^2}$$

$$= Z_{p.u.,old} \times \left(\frac{V_{b,old}}{V_{b,new}}\right)^2 \times \left(\frac{S_{b,new}}{S_{b,old}}\right)$$

$$Z_{p.u.,new} = Z_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

CONVENTION FOR PER UNIT SYSTEM:

▷ Convention for choosing base values for P.U. system is

→ Value of S_b (i.e., MVA_b) is same for the entire system

→ Ratio of V_b (i.e., kV_b) on either side of transformer is selected to be same as the ratio of transformer voltage ratings i.e., turns ratio

PER UNIT REPRESENTATION OF TRANSFORMER:

▷ A 3φ transformer forming part of a 3φ PS can be represented by a 1φ transformer in obtaining the per phase solution of the system

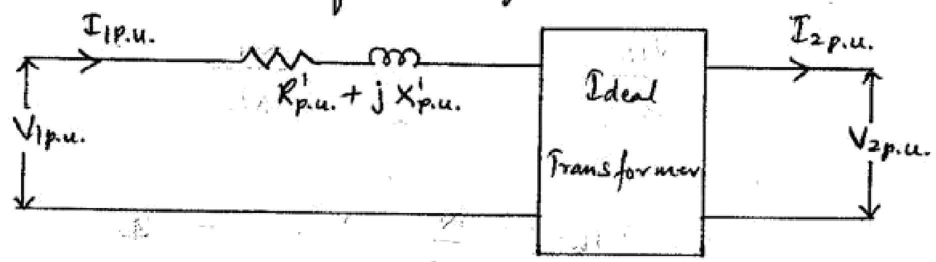


fig 9

Where → $R'_{p.u.} + j X'_{p.u.}$ is leakage impedance referred from primary side

→ $I_{1p.u.}$ & $I_{2p.u.}$ are currents in primary & secondary side

→ $V_{1p.u.}$ & $V_{2p.u.}$ are voltages in primary & secondary sides

→ Ideal transformer has ratio of $N_1 : N_2$

▷ Magnetising part of transformer is neglected because its effect is very small & neglecting it does not cause much errors

▷ Primary side voltage can be written as

$$V_{1p.u.} = \frac{V_1}{V_{1b}} \rightarrow ①$$

But W.K.T transformation ratio is

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

we have

$$V_1 = \frac{N_1}{N_2} \cdot V_2 \rightarrow (2)$$

Putting (2) in (1)

$$V_{1p.u.} = \frac{N_1}{N_2} \cdot \frac{V_2}{V_{1b}} \rightarrow (3)$$

▷ If we are choosing the two base values on both sides of transformer as

$$\frac{V_{1b}}{V_{2b}} = \frac{V_{rated1}}{V_{rated2}} = \frac{N_1}{N_2}$$

then

$$V_{1b} = \frac{N_1}{N_2} \cdot V_{2b} \longrightarrow (4)$$

putting (4) in (3)

$$V_{1p.u.} = \frac{N_1}{N_2} \cdot \frac{V_2}{\left(\frac{N_1}{N_2}\right) V_{2b}}$$

$$= \frac{N_1}{N_2} \cdot \frac{V_2}{V_{2b}} \cdot \frac{N_2}{N_1}$$

$$V_{1p.u.} = \frac{V_2}{V_{2b}} = V_{2p.u.}$$

i.e., voltage at both the sides of transformer in p.u. value are same, though actual value of the voltages will be different

Ex: For 11KV/132KV transformer if we choose base value on LV side as 11KV & on HV side as 132KV then on both sides voltage value is 1 p.u.

▷ Similarly,

$$I_{1p.u.} = \frac{I_1}{I_{1b}} \rightarrow (5)$$

W.K.T from transformation ratio

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$

$$I_1 = \frac{N_2}{N_1} \cdot I_2 \rightarrow (6)$$

Using (6) in (5)

$$I_{1p.u.} = \frac{N_2}{N_1} \cdot \frac{I_2}{I_{1b}} \rightarrow (7)$$

Writing $I_{1b} = \frac{S_b}{V_{1b}}$ (As S_b is same for entire PS we omit writing it as S_{1b})

from (4)

$$I_{1b} = \frac{S_b}{\left(\frac{N_1}{N_2} \cdot V_{2b}\right)}$$

$$= \frac{N_2}{N_1} \cdot \frac{S_b}{V_{2b}}$$

$$I_{1b} = \frac{N_2}{N_1} \cdot I_{2b} \rightarrow (8) \quad \left(\because \frac{S_b}{V_{2b}} = I_{2b}\right)$$

Using (8) in (7)

$$I_{1p.u} = \frac{N_2}{N_1} \cdot \frac{I_2}{\left(\frac{N_2}{N_1} \cdot I_{2b}\right)}$$

$$= \frac{N_2}{N_1} \cdot \frac{I_2}{I_{2b}} \cdot \frac{N_1}{N_2}$$

$$I_{1pu.} = \frac{I_2}{I_{2b}} = I_{2p.u.}$$

i.e., Current in p.u. value for I_1 & I_2 is going to be same

▷ As voltages & currents in P.U. is same on both sides of transformer the ideal transformer is simply a 1:1 transformer & can be neglected. So fig 9 reduces to fig 10 as shown

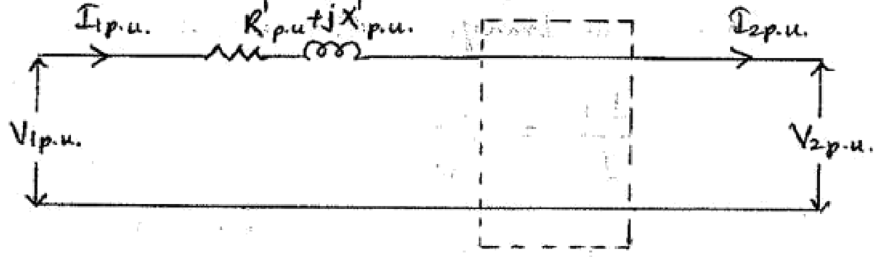


fig 10

▷ Therefore if P.U. system is used with proper base selection then the transformer is no longer required & it can be simply eliminated.

▷ Leakage Impedance can also be shown in P.U. values

$$Z_{1p.u.} = \frac{Z_1}{Z_{1b}} = \frac{Z_1}{(V_{1b}/I_{1b})} = Z_1 \cdot \frac{I_{1b}}{V_{1b}} \rightarrow (9)$$

W.K.T $Z_1 = \left(\frac{N_1}{N_2}\right)^2 Z_2 \rightarrow (10)$

$$\left[\because \frac{Z_1}{Z_2} = \frac{V_1}{I_1} \cdot \frac{I_2}{V_2} = \frac{V_1}{V_2} \cdot \frac{I_2}{I_1}\right.$$

from transformation ratio

$$= \frac{N_1}{N_2} \cdot \frac{N_1}{N_2} = \frac{N_1^2}{N_2^2}$$

$$\left.\therefore Z_1 = Z_2\left(\frac{N_1}{N_2}\right)^2\right]$$

Using (4), (8) and (10) in (9)

$$Z_{1p.u} = \frac{N_1^2}{N_2^2} \cdot Z_2 \cdot \frac{\frac{N_2}{N_1} \cdot I_{2b}}{\frac{N_1}{N_2} \cdot V_{2b}}$$

$$= \frac{N_1^2}{N_2^2} \cdot Z_2 \cdot \frac{N_2}{N_1} \cdot \frac{I_{2b}}{V_{2b}} \cdot \frac{N_2}{N_1}$$

$$= Z_2 \cdot \frac{I_{2b}}{V_{2b}}$$

$$Z_{1p.u.} = \frac{Z_2}{(V_{2b}/I_{2b})} = \frac{Z_2}{Z_{2b}} = Z_{2p.u.}$$

i.e., Even the impedance referred from both the sides of transformer in p.u. system is same

∴ Transformation ratio's for referring impedance from one side to other of transformer is no longer required.

ADVANTAGES OF PER UNIT SYSTEM:

- P.u. System gives clear idea of relative magnitude of all electrical quantities (V, P, I & Z)
- Manufacturers specify impedance of equipment in percent or p.u. on the basis of name plate rating
- P.U. impedance of equipment of same type but with widely different rating lie in a narrow range, although their impedance in Ω vary greatly with the rating

  i.e., → For most of the generators with different ratings say 10MW or 500MW, p.u. impedance value lie b/w 0.5 p.u. to 1.2 p.u. (For thermal generator it is of order of 0.8 p.u. to 0.9 p.u.), where as actual impedance value varies from few Ω to mΩ (milli ohm)

  → So while designing a system even if we do not know the exact value we can choose an average value of p.u. impedance for given equipment & perform design calculations before procuring the equipment
- P.U. impedance, voltage & current expressed with proper base is same referred to either side of a transformer. This is of great advantage as different voltage levels disappear & the entire system reduces to a system of simple impedance
- P.U. system is ideal for computerised analysis & simulation of complex PS problems
- Method of connection of transformer (Y-Y or Y-Δ etc) do not affect the P.U. impedance of the transformer
- P.u. system makes calculation relatively easier. So the actual values of PS components are converted into the P.U.

values for calculation, & after all calculation final result is obtained back in the actual values by multiplying the obtained p.u. values of the PS components with their respective base values.

SINGLE / ONE LINE DIAGRAM (SLD):

▷ As balanced 3ϕ system is always analysed on per phase basis, it is enough to show one phase & neutral in diagramatic representation of PS.

▷ Resultant diagram obtained by omitting neutral is called single line diagram (SLD)

▷ In SLD PS components are represented by standard symbols and transmission lines are represented by straight lines

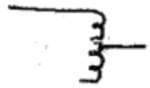
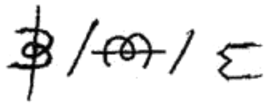
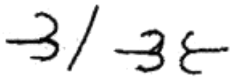
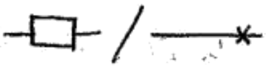
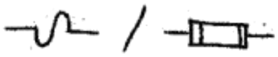
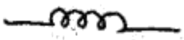
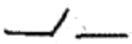
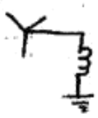
▷ SLD is defined as "A diagram showing various components of a symmetrical, balanced, 3ϕ PS by standard symbols & their interconnection through a straight line".

▷ SLDs use single line to represent all the 3 phases

▷ The purpose of SLD is to supply in short form the significant information about the system

▷ Various symbols used in SLDs are

SYMBOL	COMPONENT
	Generator/motor/rotating m/c
	2 winding power transformer
	3 winding power transformer

SYMBOL	COMPONENT
	Auto-transformer
	current transformer
	potential transformer
	power circuit Breaker (Oil / gas filled)
	Air circuit Breaker
	Fuse
	Reactor
	Lightning Arrestor
	Isolator / Disconnect switch
Δ	3φ, 3wire, delta connection
	3φ star, neutral solidly grounded
Y	3φ star, neutral ungrounded
	3φ star, neutral grounded through reactor
	3φ star, neutral grounded Through resistor
	Ammeter & Voltmeter

▷ Neutrals of synchronous machines are generally grounded through resistors or inductance coils to reduce current flow through neutral during fault. Coil so used is called ground fault neutralizer or peterson coil

BUSBAR (BUS):

▷ Buses are thick aluminium or copper bars or pipes & can be several meters long

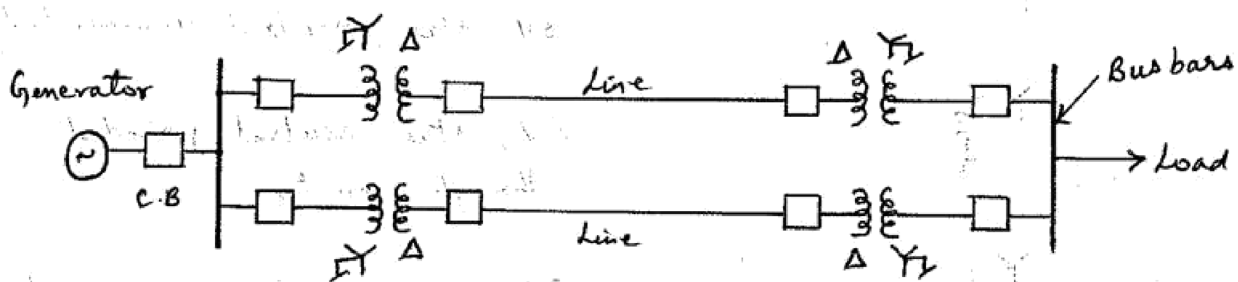
▷ They are termed as nodes in electrical circuit

▷ They are similar to transmission lines

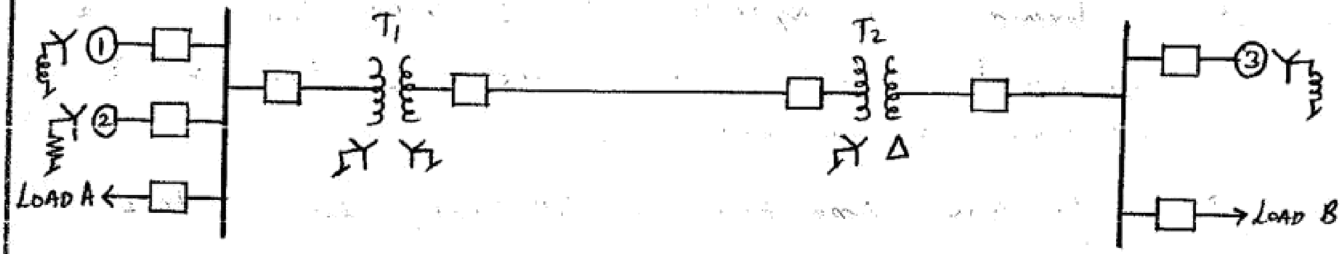
▷ Buses are required in PS because various lines which may be very thick cannot be joined by connecting them at one point. So a long bar is used & various lines & components are connected to this bar by means of nuts & bolts.
This system has been used from begining & still continues

▷ Buses in SLDs are represented as short thick straight lines perpendicular to transmission lines & to lines connecting equipment to the buses.

▷ Fig 11 shows typical examples of a SLD representation of PS



C.B = Circuit Breaker



Generator 1 : 30 MVA, 10.5 KV, $X'' = 1.6\,\Omega$

Generator 2 : 15 MVA, 6.6 KV, $X'' = 1.2\,\Omega$

Generator 3 : 25 MVA, 6.6 KV, $X'' = 0.56\,\Omega$

Transformer T1 (3ϕ) : 15 MVA, 33/11 KV, $X = 15.2\,\Omega$/phase on H.T side

Transformer T2 (3ϕ) : 15 MVA, 33/6.2 KV, $X = 16\,\Omega$/phase on H.T side

Transmission Line : $20.5\,\Omega$/phase

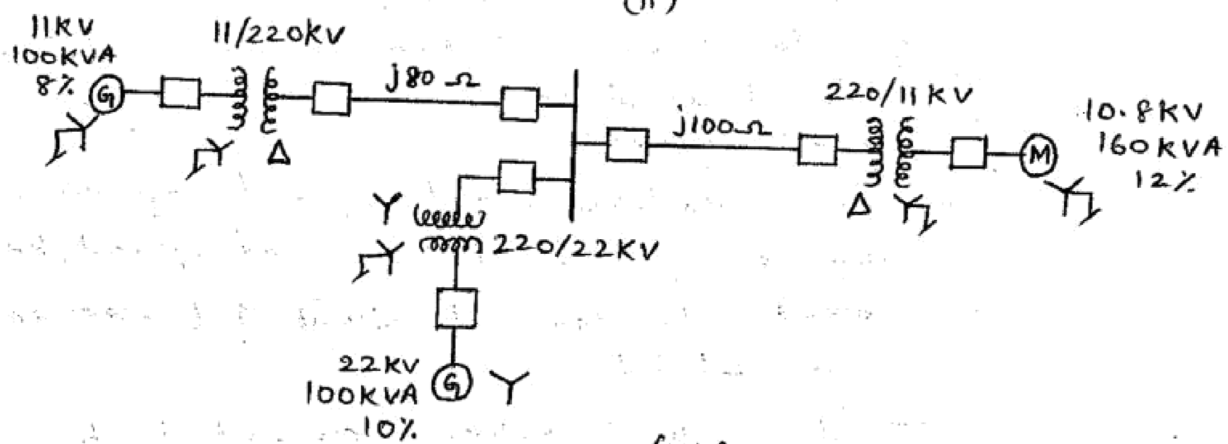
or

length = 64 Km, $X_{TL} = 0.5\,\Omega$/Km

Load A : 15 MW, 11 KV, 0.9 lagging p.f

Load B : 40 MW, 6.6 KV, 0.85 lagging p.f

(ii)



(iii)

Fig 11

▷ Generators / Motors are specified in 3ϕ MVA, line-to-line voltage & per phase reactance (equivalent star)

- ▷ Transformers are specified in 3ϕ MVA, line-to-line transformation ratio & per phase (equivalent star) impedance on one side
- ▷ Loads are specified in 3ϕ MW, line-to-line Voltage & p.f
- ▷ Transmission lines are specified in impedance/phase or length in Km & impedance/Km

IMPEDANCE AND REACTANCE DIAGRAMS:

▷ The SLD is used to draw a 1ϕ equivalent circuit of the system in order to calculate the performance of system under load conditions or on occurance of short circuit i.e., on occurance of fault

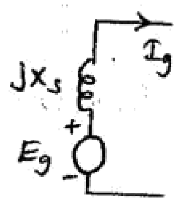
(i) IMPEDANCE DIAGRAM:

- ▷ It is used for load flow studies
- ▷ Obtained by replacing each component of PS by its 1ϕ equivalent circuit
- ▷ APPROXIMATION TO FORM IMPEDANCE DIAGRAM:
  - → Current limiting impedances connected between generator neutral and ground are neglected since under balanced conditions no current flows through neutral
  - → Since magnetising current of a transformer is very low compared to full-load current, shunt branches in equivalent circuit of transformer are neglected
  - → If inductive reactance compared to the resistance is very high then resistances are omitted. This may introduce little error but results may be satisfactory

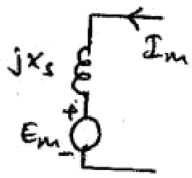
(ii) Reactance Diagram:

- It is used for fault calculations
- Obtained after omitting all resistances, magnetising circuit of Transformer & capacitance of transmission line in impedance diagram
- **Approximation to form Reactance Diagram (RD):**
  - → All static loads are neglected because they have little effect on total line current during fault
  - → Synchronous motor loads are always included in making fault calculations since generated emf contribute to short circuit current
  - → Induction Motor (Emf in series with inductive reactance) is considered only if diagram is used to determine current immediately after fault has occured
  - → Normally induction motors are ignored in computing current a few cycles after fault occurs because current contributed by induction motor dies out very quickly after induction motor is short circuited
- **Representation of PS Components in RD:**

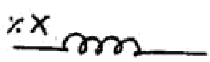
3ϕ Generator



Synchronous Motor



Transformer

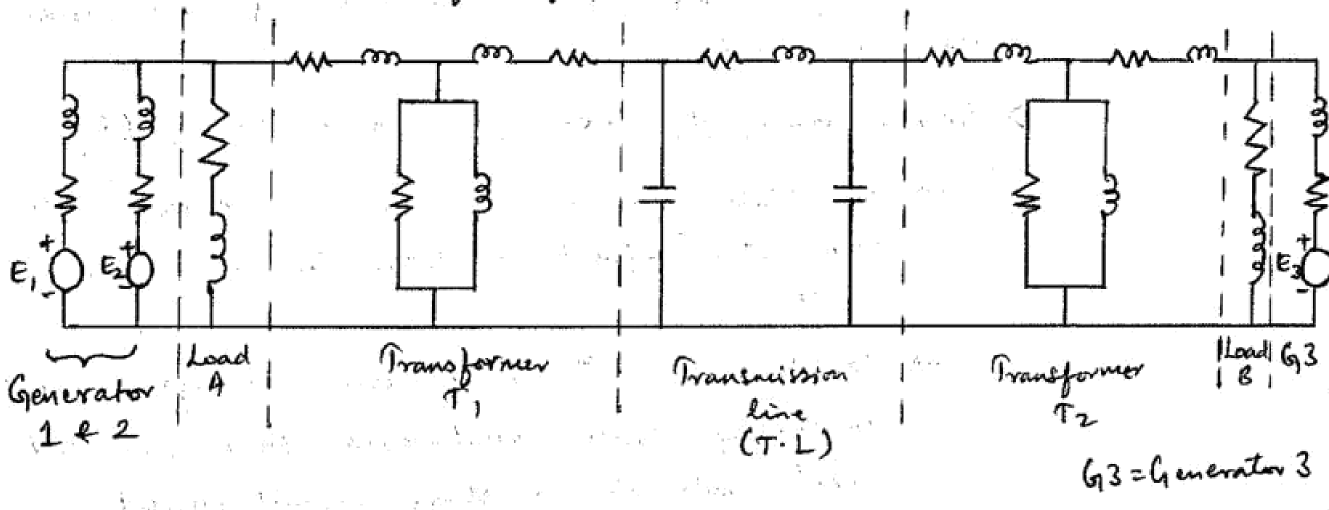


Transmission Line

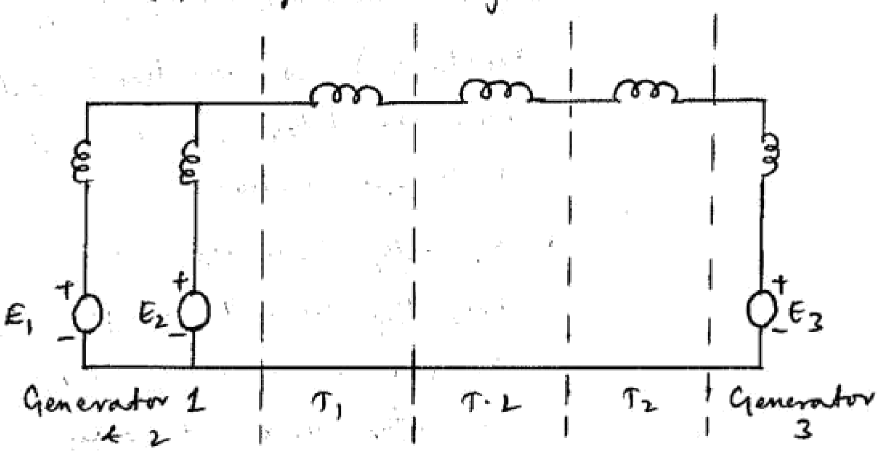
Inductive Reactance

▷ Impedance & reactance diagrams are sometimes called positive sequence diagrams since they show impedances to balanced currents in a symmetrical 3ϕ system.

▷ Consider SLD of fig 11 (ii)



(a) Impedance Diagram



(b) Reactance Diagram

fig 12

PROCEDURE TO FORM REACTANCE DIAGRAM FROM SLD:

▷ A base kilovolt KV_b & a base kilovolt Amper or Megavolt Ampere KVA_b or MVA_b is selected in one part of the system

▷ KVA_b or MVA_b remains same for all section of PS

▷ Base values for 3ϕ PS are line-to-line KV_b & 3ϕ MVA_b or 3ϕ KVA_b

▷ For other part of the system i.e., on other side of transfo-rmer KV_b for each part is determined according to line-to-line voltage ratios of transformer

$$KV_b \text{ on LT section} = KV_b \text{ on HT section} \times \frac{LT\ VR}{HT\ VR}$$

$$KV_b \text{ on HT section} = KV_b \text{ on LT section} \times \frac{HT\ VR}{LT\ VR}$$

Wher VR = Voltage Rating

▷ It is helpful to mark KV_b of each part of the system on SLD

▷ Impedance information for 3ϕ transformer is in ohm (actual impedance), or in per unit or percent of the base determined by the ratings

▷ → In Reactance diagram, reactance of all components of PS are expressed in Common base

→ Then the reactance of each component is converted to p.u. reactance on selected new base value

→ If specified reactance of component is in ohm then

$$\text{p.u. reactance} = \frac{\text{Actual reactance in } \Omega}{\text{Base impedance in } \Omega}$$

→ If specified reactance of component is in p.u. on components rating as base value, then component rating is considered as old base value & selected base value is considered as new base value & the p.u. reactance is calculated from the formula

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

▷ For three single phase transformers connected as a 3ϕ unit the 3ϕ ratings are determined from the 1ϕ rating of each individual transformer. Impedance in percent for the 3ϕ unit is same as that for each individual transformer.

NODE EQUATIONS AND BUS ADMITTANCE MATRIX (Y_{bus}):

▷ The junction formed when two or more pure elements (R, L, C or an ideal voltage or current source) are connected to each other at their terminals are called nodes

▷ In PS network (PS represented by its impedance or reactance diagram is called PS network or PS circuit) buses are treated as nodes and their voltages are obtained from conventional nodal analysis technique

NOTE: Bus / Bus bars are thick copper or aluminium conductors having negligible resistance. Hence they have zero voltage drop when conducting rated currents. Therefore buses are considered as points of constant voltage in PS

▷ Consider SLD and Reactance Diagram of a simple PS as shown in fig 13 (a) & (b)

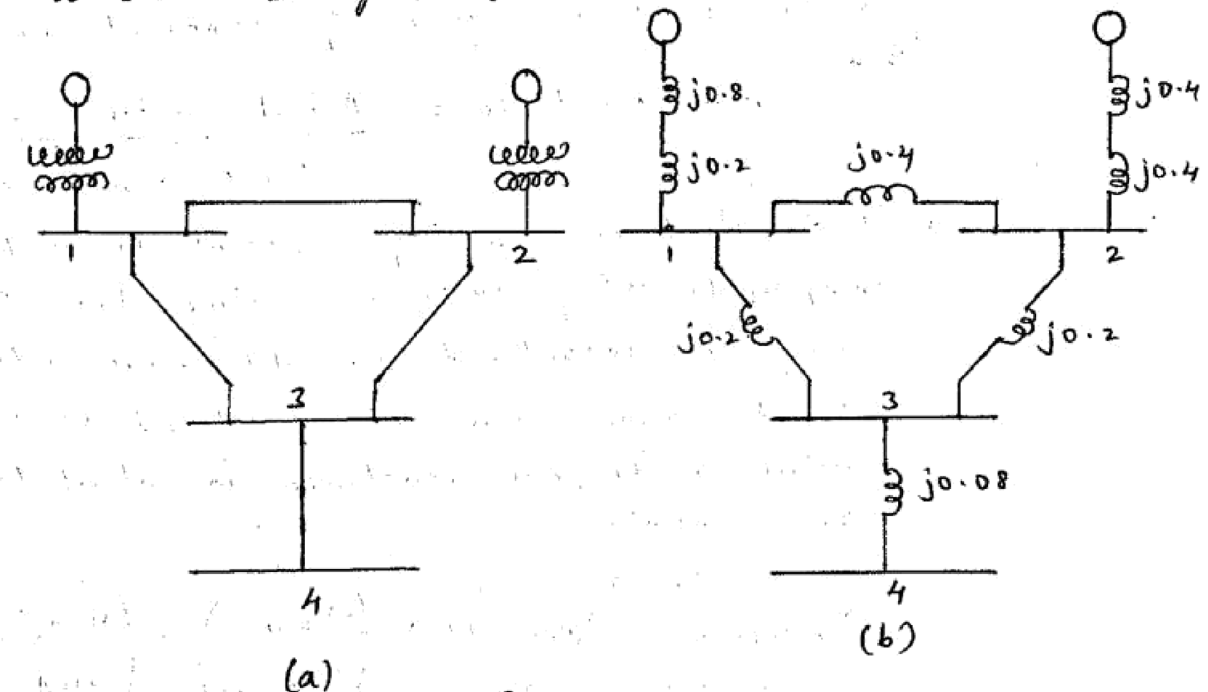
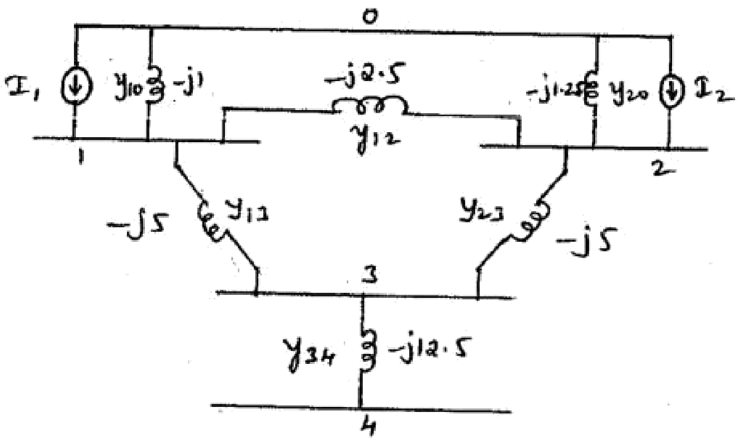


Fig 13



(c)

fig 13

- To obtain node voltage equations all impedances are converted into admittances, since nodal solution is based on KCL
- Admittance diagram is shown in fig 13 (c)
- Node 0 (which is normally ground) is taken as reference

  NOTE: If direction of current in current source is towards the node then source is considered positive, and if it is away from the node then source is considered negative

- Applying KCL to independent nodes 1, 2, 3 & 4 we get

AT NODE 1:

$$I_1 = y_{10}V_1 + y_{12}(V_1 - V_2) + y_{13}(V_1 - V_3)$$
$$= y_{10}V_1 + y_{12}V_1 - y_{12}V_2 + y_{13}V_1 - y_{13}V_3$$
$$I_1 = (y_{10} + y_{12} + y_{13})V_1 - y_{12}V_2 - y_{13}V_3 \rightarrow ①$$

AT NODE 2:

$$I_2 = y_{20}V_2 + y_{12}(V_2 - V_1) + y_{23}(V_2 - V_3)$$
$$= y_{20}V_2 + y_{12}V_2 - y_{12}V_1 + y_{23}V_2 - y_{23}V_3$$
$$I_2 = -y_{12}V_1 + (y_{20} + y_{12} + y_{23})V_2 - y_{23}V_3 \rightarrow ②$$

AT NODE 3:

$$0 = y_{13}(V_3 - V_1) + y_{23}(V_3 - V_2) + y_{34}(V_3 - V_4)$$
$$= y_{13}V_3 - y_{13}V_1 + y_{23}V_3 - y_{23}V_2 + y_{34}V_3 - y_{34}V_4$$
$$0 = -y_{13}V_1 - y_{23}V_2 + (y_{13} + y_{23} + y_{34})V_3 - y_{34}V_4 \rightarrow ③$$

AT NODE 4:

$$0 = y_{34}(V_4 - V_3)$$

$$0 = -y_{34}V_3 + y_{34}V_4 \rightarrow (4)$$

▷ Let

$$y_{10} + y_{12} + y_{13} = Y_{11} \qquad -y_{12} = Y_{12} = Y_{21}$$

$$y_{20} + y_{12} + y_{23} = Y_{22} \qquad -y_{13} = Y_{13} = Y_{31}$$

$$y_{13} + y_{23} + y_{34} = Y_{33} \qquad -y_{23} = Y_{23} = Y_{32}$$

$$y_{34} = Y_{44} \qquad -y_{34} = Y_{34} = Y_{43}$$

Then node equations (1), (2), (3) and (4) can be written in general as

$$I_1 = Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 + Y_{14}V_4 \rightarrow (5)$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4 \rightarrow (6)$$

$$I_3 = Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3 + Y_{34}V_4 \rightarrow (7)$$

$$I_4 = Y_{41}V_1 + Y_{42}V_2 + Y_{43}V_3 + Y_{44}V_4 \rightarrow (8)$$

▷ In fig 13 (c) $Y_{14} = Y_{41} = 0$ & $Y_{24} = Y_{42} = 0$ since there are no connection b/w buses 1 & 4 and 2 & 4

▷ Writing the independent equations (5), (6), (7) & (8) in matrix form

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

▷ For n independent buses node equation in matrix form is

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \vdots & \vdots & & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}$$

or $$I = Y_{bus} \cdot V$$

where Y_{bus} = Bus Admittance Matrix of order $(n \times n)$
(n is no of independent nodes excluding reference node. ∴ Total no of nodes are $n+1$)
V = Vector of node/bus voltage and is a matrix of order $(n \times 1)$

I = Vector of injected node currents and is a matrix of order $(n \times 1)$

y = Branch admittance & is referred to as primitive admittance

- Diagonal elements of each node, Y_{ii} $(i = 1, 2, --- n)$, is sum of all primitive admittances connected to it. They are called as self admittance or driving point admittance

$$\text{i.e.,} \quad Y_{ii} = \sum_{j=0}^{n} y_{ij} \quad (j \neq i)$$

- Off diagonal elements, Y_{ij} $(i = j = 1, 2, --- n)$, is equal to negative of primitive admittance connected b/w nodes i and j, and is called mutual admittance or transfer admittance

$$\text{i.e.,} \quad Y_{ij} = Y_{ji} = -y_{ij}$$

- Therefore Y_{bus} is a symmetric square matrix (it is symmetric about principle diagonal). Also $Y_{ij} = 0$ if buses i and j are not connected

- In large ps network each bus is connected to a few (say 3 or 4) nearby buses. Consequently many off diagonal elements are zero. Such a matrix is called Sparse matrix.

- Sparse matrix will have only around 2% of non-zero elements.

- In Large ps with hundred's of buses sparsity may be as high as 90%.

- This will lead to higher computational speed and also storage and reduction of round off errors

- Bus admittance matrix (Y_{bus}) is often used in solving load flow problems

<u>NOTE:</u> Y_{bus} obtained with one bus as reference is non singular else it is singular

Steps/Rules To Construct Y_{bus} By Inspection:

- Y_{bus} is symmetric
- Y_{ii}, the self admittance (diagonal elements of each node) is equal to the sum of all primitive admittances connected to it
- Y_{ij}, the mutual admittance (off diagonal elements) is equal to negative of primitive admittance b/w nodes i & j
- If more than one component is in series in a line connected to bus or if more than one component is connected in parallel b/w two buses then equivalent primitive admittance is obtained before determining Y_{ii} and Y_{ij}

[NOTE: $I = Y_{bus} \cdot V$

multiplying Y_{bus}^{-1} on both sides we get

$$I \cdot Y_{bus}^{-1} = Y_{bus} \cdot V \cdot Y_{bus}^{-1}$$

$$\therefore V = Y_{bus}^{-1} \cdot I$$

$$\therefore V = Z_{bus} \cdot I$$

Where Z_{bus} is termed as Bus Impedance Matrix]

REPRESENTATION OF POWER SYSTEM COMPONENTS

PROBLEMS:

1. Calculate the per unit impedance of a synchronous motor rated at 200 KVA, 13.2 KV and having a reactance of 50 Ω/phase

Sol Synchronous motor rating itself is choosen as base KV & base KVA

$$\therefore \quad KVA_b = 200 \text{ KVA}$$

$$KV_b = 13.2 \text{ KV}$$

$$Z = X = 50 \ \Omega/\text{phase}$$

$$Z_{p.u.} = \frac{Z \times KVA_b}{KV_b^2 \times 1000}$$

$$= \frac{50 \times 200}{(13.2)^2 \times 1000}$$

$$= 0.0574 \text{ p.u.}$$

2. A 3φ, Δ-Y transformer with rating 100 KVA, 11 KV/400 V has its primary and secondary leakage reactance as 12 Ω/phase and 0.05 Ω/phase respectively. Calculate the p.u. reactance of transformer

Sol CASE 1: P.U. reactance of transformer referred to primary

Primary rating of transformer are choosen as base KVA & base KV

$$\therefore \quad KVA_b = 100 \text{ KVA}$$

$$KV_b = 11 \text{ KV}$$

Voltage transformation ratio $K = \dfrac{V_2}{V_1}$

$$K = \frac{400}{11 \times 10^3} = 0.0363$$

Total leakage reactance referred from primary $X_{01} = X_1 + X_2'$

$$= X_1 + \frac{X_2}{K^2}$$

$$= 12 + \frac{0.05}{(0.0363)^2}$$

$$= 49.94 \ \Omega/\text{phase}$$

$$\text{P.U. reactance} = \frac{\text{Actual reactance in } \Omega}{\text{Base impedance in } \Omega}$$

$$(X_{01})_{p.u.} = \frac{X_{01}}{Z_b}$$

W.K.T $Z_b = \frac{KV_b^2 \times 1000}{KVA_b} \ \Omega$

$$\therefore (X_{01})_{p.u.} = \frac{X_{01} \times KVA_b}{KV_b^2 \times 1000}$$

$$= \frac{49.94 \times 100}{(11)^2 \times 1000}$$

$$= 0.0412 \text{ p.u.}$$

CASE 2: P.U. reactance of transformer referred to secondary

Secondary rating of transformer are choosen as base KVA & base KV

$$\therefore KVA_b = 100 \text{ KVA}$$

$$KV_b = 400 \text{ V} = 0.4 \text{ KV}$$

Voltage transformation ratio $K = \frac{400}{11 \times 10^3} = 0.0363$

Total leakage reactance referred from secondary $X_{02} = X_1' + X_2$

$$X_{02} = K^2 X_1 + X_2$$

$$= (0.0363)^2 \times 12 + 0.05$$

$$= 0.0658 \ \Omega/\text{phase}$$

$$(X_{02})_{p.u.} = \frac{X_{02} \times KVA_b}{KV_b^2 \times 1000}$$

$$= \frac{0.0658 \times 100}{(0.4)^2 \times 1000}$$

$$= 0.0411 \text{ p.u.}$$

$\therefore$ It is observed that p.u. reactance of a transformer referred to primary & secondary are same

3. A 3ϕ, $Y-\Delta$ transformer is constructed using 3 identical 1ϕ transformers of rating 200KVA, 63.51 KV/11KV. The impedances of primary & secondary are $20+j45 \ \Omega$ and $0.1+j0.2 \ \Omega$ respectively. Calculate the p.u. impedance of the transformer

Sol The 3ϕ transformer is formed using 3 identical 1ϕ transformers. Hence KVA rating of 3ϕ transformer is 3 times that of 1ϕ transformer

$\therefore$ KVA rating of 3ϕ transformer $= 3 \times 200$
$= 600\text{KVA}$

Line voltage rating of $Y-\Delta$ transformer
$= 63.51 \times \sqrt{3}$ KV/11KV
$= 110\text{KV}/11\text{KV}$

NOTE: Primary voltage is multiplied with $\sqrt{3}$ because Primary is star connected & in star $V_L = \sqrt{3} V_p$

CASE 1:

Primary rating is choosen as base value

$\therefore \ KVA_b = 600 \ KVA$

$KV_b = 110 \ KV$

Voltage transformation ratio $k = \frac{V_2}{V_1} = \frac{11 \times 10^3}{110 \times 10^3}$

$\therefore k = 0.1$

Total impedance referred to primary $Z_{01} = Z_1 + Z_2'$

$= Z_1 + \frac{Z_2}{k^2}$

$= (20 + j45) + \frac{(0.1 + j0.2)}{(0.1)^2}$

$= (20 + j45) + (10 + j20)$

$= 30 + j65 \ \Omega/\text{phase}$

$$Z_{p.u.} = \frac{Z \times KVA_b}{KV_b^2 \times 1000}$$

$$= \frac{(30 + j65) \times 600}{(110)^2 \times 1000}$$

$= 1.48 \times 10^{-3} + j \, 3.22 \times 10^{-3}$ p.u.

$= 0.00148 + j0.00322$ p.u.

CASE 2:

Secondary rating is choosen as base value

$\therefore \ KVA_b = 600 \ KVA$

$KV_b = 11 \ KV$

Voltage transformation ratio $k = \frac{11 \times 10^3}{110 \times 10^3} = 0.1$

Total impedance referred to secondary $Z_{02} = Z_1' + Z_2$

$= k^2 Z_1 + Z_2$

$$= (0.1)^2 \times (20+j45) + (0.1+j0.2)$$

$$= (0.2+j0.45) + (0.1+j0.2)$$

$$= 0.3+j0.65 \ \Omega/\text{phase}$$

$$Z_{p.u.} = \frac{Z \times KVA_b}{KV_b^2 \times 1000}$$

$$= \frac{(0.3+j0.65)\times 600}{(11)^2 \times 1000}$$

$$= 1.48\times10^{-3} + j\,3.22\times10^{-3} \text{ p.u.}$$

$$= 0.00148 + j\,0.00322 \text{ p.u.}$$

4. A 50KW, 3ϕ, Y connected load is fed by a 200KVA transformer with voltage rating 11KV/400V through a feeder. The length of the feeder is 0.5 Km & its impedance is (0.1 +j0.2) Ω/Km. If load p.f is 0.8, Calculate the p.u. impedance of load & feeder.

Sol Let us consider secondary rating of transformer as base value

[$\because$ if we consider primary rating of transformer as base value then the base voltage rating on secondary will be 400V itself

i.e., KV_b on LT side = KV_b on HT side $\times \frac{LT\ VR}{HT\ VR}$

where VR = voltage rating

KV_b on LT Side i.e., 400V $= 11\text{KV} \times \frac{400}{11\text{KV}}$

$= 400$ V]

$\therefore$ $KVA_b = 200$ KVA

$KV_b = 400V = 0.4KV$

feeder impedance is given as $(0.1 + j0.2)\ \Omega/km$

& length of feeder is 0.5 km

$\therefore$ Actual feeder impedance $Z_{fed} = (0.1+j0.2) \times 0.5$

$$= 0.05 + j0.1\ \Omega/\text{phase}$$

$$Z_{p.u.,fed} = \frac{Z_{fed} \times KVA_b}{KV_b^2 \times 1000}$$

$$= \frac{(0.05+j0.1) \times 200}{(0.4)^2 \times 1000}$$

$$= 0.0625 + j0.125\ p.u.$$

Given $P = 50\ KW$

$\cos\phi = 0.8$

$\phi = \cos^{-1}(0.8) = 36.87^\circ$

$\sin\phi = \sin 36.87 = 0.6$

Reactive power, $Q = \frac{P}{\cos\phi} \times \sin\phi$ $\left[\because Q = 3V_pI_p\sin\phi \right.$ since $P = 3V_pI_p\cos\phi$

$Q = \frac{P}{\cos\phi} \times \sin\phi = \frac{3V_pI_p\cos\phi}{\cos\phi} \times \sin\phi$

$\left. = 3V_pI_p\sin\phi \right]$

$$= \frac{50}{0.8} \times 0.6$$

$$= 37.5\ KVAr$$

W.K.T if the 3ϕ load is a star connected load then the load impedance per phase is given by

$$Z_L = \frac{|V_L|^2}{P - jQ} = \frac{(400)^2}{50\times10^3 - j37.5\times10^3}$$

$$= \frac{(400)^2}{(50 - j37.5)\times10^3}$$

$$= 2.048 + j1.536\ \Omega/\text{phase}$$

$$Z_{p.u,L} = \frac{Z_L \times KVA_b}{KV_b^2 \times 1000} = \frac{(2.048+j1.536)\times 200}{(0.4)^2\times1000} = 2.56 + j1.92\ p.u.$$

5. The 3ϕ ratings of a three winding transformer are

Primary : Y-connected, 110KV, 20MVA

Secondary : Y-Connected, 13.2KV, 15MVA

Tertiary : Δ-Connected, 2.1KV, 0.5MVA

Three short circuit tests performed on this transformer yielded the following results

(i) Primary excited, secondary shorted : 2290V, 52.5A
(ii) Primary excited, Tertiary shorted : 1785V, 52.5A
(iii) Secondary excited, tertiary shorted : 148V, 328A

find the p.u. impedances of the star connected 1ϕ equivalent circuit for a base of 20MVA, 110KV in the primary circuit. Neglect resistances.

Sol

$$MVA_{b,p} = 20MVA$$

$$KV_{b,p} = 110KV$$

from test (i)

$$Z_{ps} = \frac{2290/\sqrt{3}}{52.5} = 25.18\ \Omega/\text{phase}$$

NOTE: Impedance value is generally calculated by using phase values of voltage & current. The given voltage is the line voltage & since primary is star connected we have $V_p = V_L/\sqrt{3}$

$$(Z_{ps})_{p.u.} = \frac{Z_{ps}}{Z_{b,p}} = \frac{Z_{ps} \times MVA_{b,p}}{KV_{b,p}^2}$$

$$= \frac{25.18 \times 20}{(110)^2}$$

$$= 0.0416\ \text{p.u.}$$

from test (ii)

$$Z_{pt} = \frac{1785/\sqrt{3}}{52.5} = 19.63\ \Omega/\text{phase}$$

$$(Z_{pt})_{p.u.} = \frac{Z_{pt}}{Z_{b,p}} = \frac{Z_{pt} \times MVA_{b,p}}{KV_{b,p}^2}$$

$$= \frac{19.63 \times 20}{(110)^2}$$

$$= 0.0324 \text{ p.u.}$$

from test (iii)

$$Z_{st} = \frac{148/\sqrt{3}}{328} = 0.26\ \Omega/\text{phase}$$

Base MVA on secondary remains same

i.e., $MVA_{b,s} = 20 MVA$

Base KV on secondary side is calculated from the formula

$$KV_b \text{ on LT side} = KV_b \text{ on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

$$KV_{b,s} = KV_{b,p} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

$$= 110 \times \frac{13.2}{110}$$

$$= 13.2 \text{ KV}$$

$$(Z_{st})_{p.u.} = \frac{Z_{st}}{Z_{b,s}} = \frac{Z_{st} \times MVA_{b,s}}{KV_{b,s}^2}$$

$$= \frac{0.26 \times 20}{(13.2)^2}$$

$$= 0.0298 \text{ p.u.}$$

$$\therefore (Z_p)_{p.u.} = \frac{1}{2}\left[(Z_{ps})_{p.u.} + (Z_{pt})_{p.u.} - (Z_{st})_{p.u.}\right]$$

$$= \frac{1}{2}\left[0.0416 + 0.0324 - 0.0298\right] = 0.0221 \text{ p.u.}$$

$$(Z_s)_{p.u.} = \frac{1}{2}\left[(Z_{ps})_{p.u.} + (Z_{st})_{p.u.} - (Z_{pt})_{p.u.}\right]$$

$$= \frac{1}{2}\left[0.0416 + 0.0298 - 0.0324\right]$$

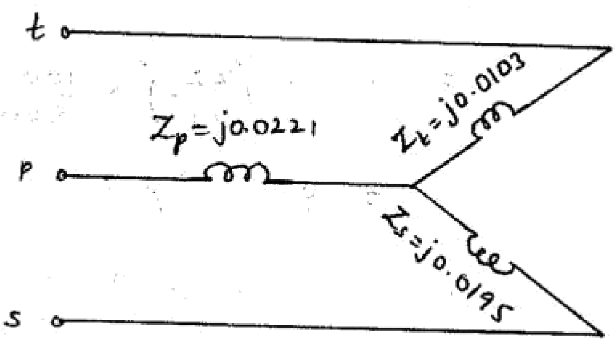
$$= 0.0195 \text{ p.u.}$$

$$(Z_t)_{p.u.} = \frac{1}{2}\left[(Z_{pt})_{p.u.} + (Z_{st})_{p.u.} - (Z_{ps})_{p.u.}\right]$$

$$= \frac{1}{2}\left[0.0324 + 0.0298 - 0.0416\right]$$

$$= 0.0103 \text{ p.u.}$$

Hence the 1ϕ star connected equivalent circuit of 3 winding transformer is as shown (Resistances are neglected)



6. a) A generator is rated 500MVA, 22 KV. Its Y-connected winding has a reactance of 1.1 p.u. find the ohmic value of the reactance of winding

b) If the generator is working in a circuit for which the bases are specified as 100MVA, 20KV. Then find the p.u. value of reactance of generator winding on the specified base

Sol (a) Generator p.u. reactance is specified by taking its rating as base values

$$\therefore MVA_b = 500 MVA$$

$$KV_b = 22 KV$$

$$X_{p.u.} = \frac{\text{Actual reactance in } \Omega}{\text{Base impedance in } \Omega} = \frac{X}{Z_b}$$

$$X = Z_b \times X_{p.u.} = X_{p.u.} \times \frac{KV_b^2}{MVA_b}$$

$$= 1.1 \times \frac{(22)^2}{500}$$

$$= 1.0648 \ \Omega/\text{phase}$$

(b) New base values are

$$MVA_{b,new} = 100 \ MVA$$

$$KV_{b,new} = 20 \ KV$$

Old base values are

$$MVA_{b,old} = 500 \ MVA$$

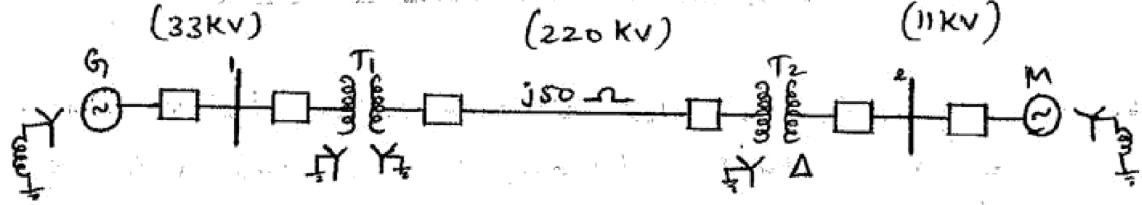
$$KV_{b,old} = 22 \ KV$$

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$$

$$= 1.1 \times \left(\frac{22}{20}\right)^2 \times \frac{100}{500}$$

$$= 0.2662 \ p.u.$$

7. Draw the p.u. reactance diagram for the power system as shown



The rating of the generator, Motor and transformers are:

Generator : 40MVA, 25KV, $X'' = 20\%$

Motor : 50MVA, 11KV, $X'' = 30\%$

Y-Y transformer: 40 MVA, 33Y/220Y KV, $X = 15\%$

Y-Δ transformer: 30MVA, 11Δ/220Y KV, $X = 15\%$

Use a base of 100MVA, 220KV in 50 Ω line.

(MAY/JUNE 2010 – 8M

JUNE/JULY 2011 – 10M

JUNE/JULY 2008 – 8M)

Sol Given $MVA_b = 100MVA$ (This is same for entire system)

In j50 Ω line $KV_b = 220 KV$

KV_b in Generator circuit $= 220 \times \frac{33}{220}$

$$KV_b = 33\,KV$$

KV_b in Motor circuit $= 220 \times \frac{11}{220}$

$$KV_b = 11\,KV$$

[These KV_b values are obtained from the formula

KV_b on LT side = KV_b on HT side × $\frac{\text{LT voltage rating}}{\text{HT voltage rating}}$]

These values are used as new base values for Calculation of p.u. reactance of different Components. It is very helpful to mark the KV_b of each part of the system on SLD (as shown in paranthesis on the SLD)

REACTANCE OF GENERATOR G_1:

$$X_{p.u.,\,new} = X_{p.u.,\,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.2 \times \left(\frac{25}{33}\right)^2 \times \left(\frac{100}{40}\right)$$

$$= 0.287\,p.u.$$

REACTANCE OF TRANSFORMER T_1:

$$X_{p.u.,\,new} = X_{p.u.,\,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.15 \times \left(\frac{33}{33}\right)^2 \times \left(\frac{100}{40}\right)$$

$$= 0.375\,p.u.$$

NOTE: Generally the new p.u. impedance/reactance of transformer is calculated by selecting $KV_{b,old}$ + $KV_{b,new}$ of the primary side or of the secondary side

REACTANCE OF TRANSMISSION LINE:

$$X_{p.u.} = \frac{X \times MVA_b}{KV_b^2}$$

$$= \frac{50 \times 100}{(220)^2}$$

$$= 0.1033 \text{ p.u.}$$

REACTANCE OF TRANSFORMER T2:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.15 \times \left(\frac{220}{220}\right)^2 \times \left(\frac{100}{30}\right)$$

$$= 0.5 \text{ p.u.}$$

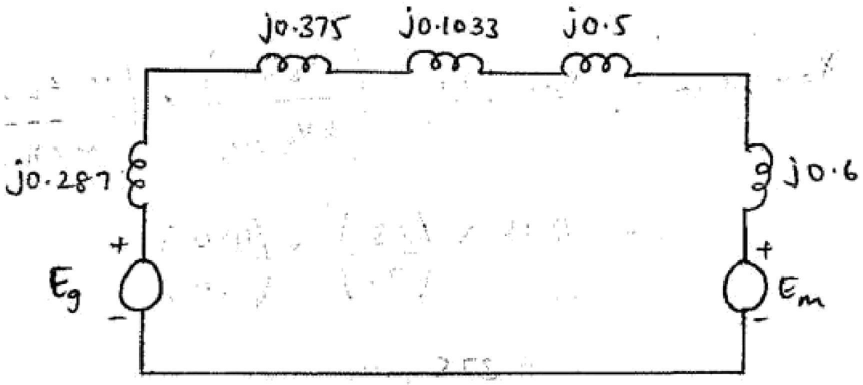
REACTANCE OF MOTOR:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

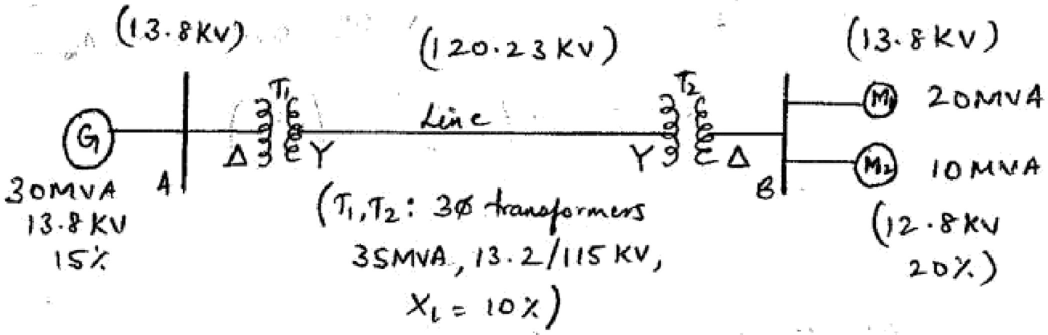
$$= 0.3 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{100}{50}\right)$$

$$= 0.6 \text{ p.u.}$$

REACTANCE DIAGRAM:



8. Draw the p.u. reactance diagram for the system shown. choose a base of 30MVA, 13.8KV in the generator circuit, assume the line reactance to be 80 Ω



(JULY 2007 – 8M)

Sol

$MVA_b = 30\,MVA$ (Same for entire system)

In generator circuit $KV_b = 13.8\,KV$

In 80Ω transmission line $KV_b = 13.8 \times \frac{115}{13.2}$

$= 120.23\,KV$

In motor circuit $KV_b = 120.23 \times \frac{13.2}{115}$

$= 13.8\,KV$

These values are new base values for calculating p.u. reactance of different components

<u>REACTANCE OF GENERATOR G:</u>

Since Generator rating & base values are same, its p.u. reactance remains same

$\therefore X_{p.u.,new} = 0.15\,p.u.$

i.e., $X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$

$= 0.15 \times \left(\frac{13.8}{13.8}\right)^2 \times \left(\frac{30}{30}\right)$

$= 0.15\,p.u.$

REACTANCE OF TRANSFORMERS T_1 & T_2: Since rating of both transformers are same

$$(X_{p.u,new})_{T_1} = (X_{p.u.,new})_{T_2} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.1 \times \left(\frac{13.2}{13.8}\right)^2 \times \left(\frac{30}{35}\right)$$

$$= 0.0784 \, p.u.$$

REACTANCE OF TRANSMISSION LINE

$$X_{p.u.} = \frac{X \times MVA_b}{KV_b^2}$$

$$= \frac{80 \times 30}{(120.23)^2}$$

$$= 0.166 \, p.u.$$

REACTANCE OF MOTOR M1:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.2 \times \left(\frac{12.8}{13.8}\right)^2 \times \left(\frac{30}{20}\right)$$

$$= 0.258 \, p.u.$$

REACTANCE OF MOTOR M2:

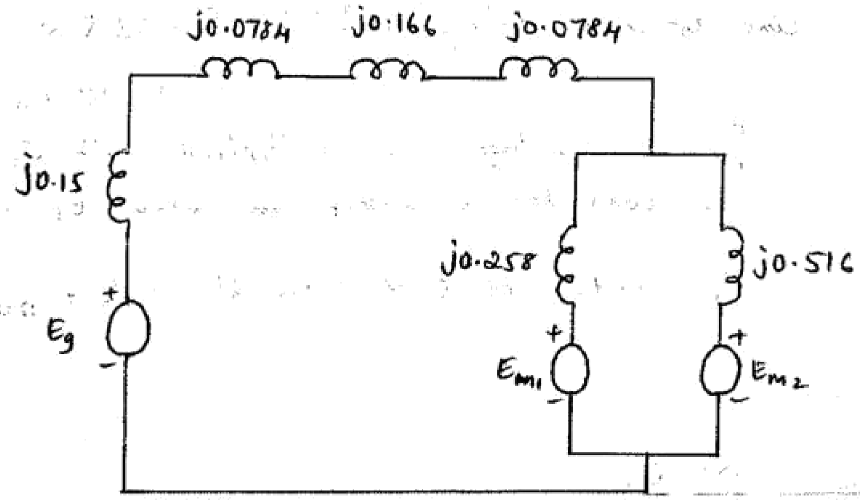
$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.2 \times \left(\frac{12.8}{13.8}\right)^2 \times \left(\frac{30}{10}\right)$$

$$= 0.516 \, p.u$$

REACTANCE DIAGRAM:

The reactance diagram is as shown



9. Figure shows SLD of PS for which details of system components are as follows

G : Synchronous Generator, 80MVA, 6KV, $x_d'' = 8\%$

T_1 : 3φ transformer, 85 MVA, 110KV(Y) / 6.6KV(Δ), x=10%

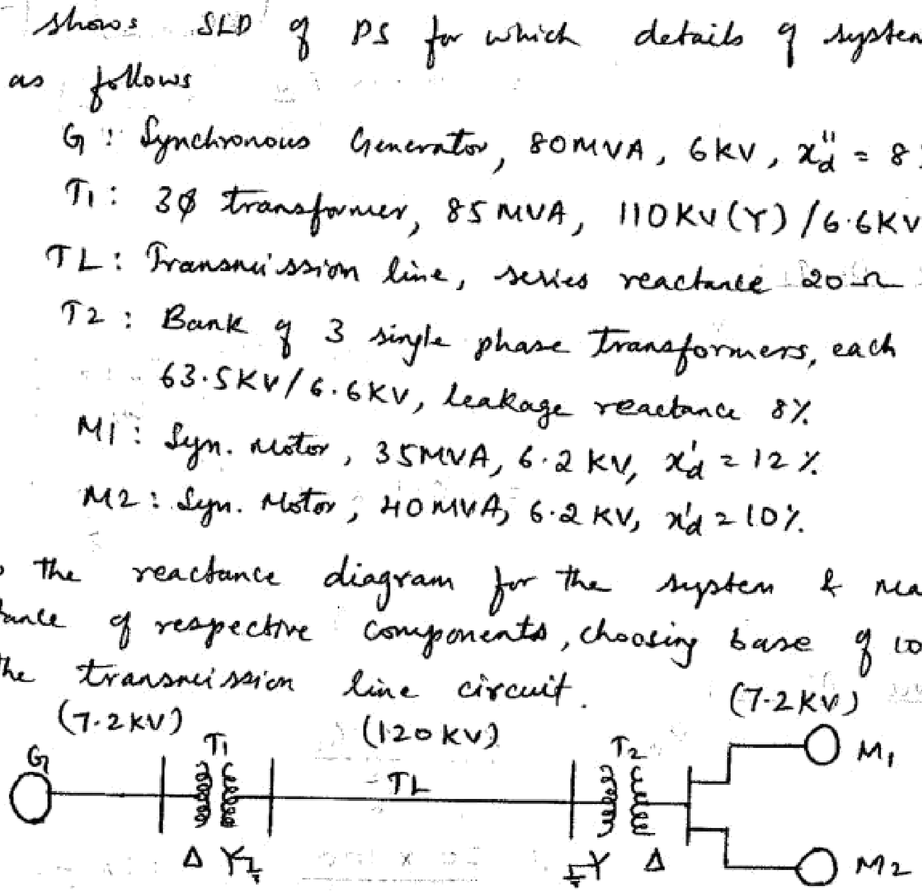
TL : Transmission line, series reactance 20Ω

T2 : Bank of 3 single phase transformers, each rated 30MVA, 63.5KV/6.6KV, leakage reactance 8%

M1 : Syn. motor, 35MVA, 6.2 KV, $x_d' = 12\%$

M2 : Syn. Motor, 40MVA, 6.2 KV, $x_d' = 10\%$

Draw the reactance diagram for the system & mark the p.u. reactance of respective components, choosing base of 100MVA, 120KV in the transmission line circuit.



(JULY/AUGUST 2005 - 12 M)

Sol

$MVA_b = 100MVA$ (Same for entire system)

In TL circuit $KV_b = 120KV$

In G circuit $KV_b = 120 \times \frac{6.6}{110} = 7.2\ KV$

Transformer T_2 is a bank of three 1φ transformers
∴ 3φ rating of transformer T_2 is

MVA rating : $3 \times 30MVA = 90MVA$

Line-to-line voltage ratio : $\sqrt{3} \times 63.5$ KV / 6.6 KV

$= 110$ KV / 6.6 KV

[Primary voltage is multiplied with $\sqrt{3}$ because it is star connected & w.k.t in star $E_L = \sqrt{3}\, E_p$]

In motor (M_1 & M_2) circuit $KV_b = 120 \times \frac{6.6}{110}$

$= 7.2$ KV

<u>REACTANCE OF G</u>:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.08 \times \left(\frac{6}{7.2}\right)^2 \times \left(\frac{100}{80}\right)$$

$$= 0.0694 \text{ p.u.}$$

<u>REACTANCE OF T_1</u>:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.1 \times \left(\frac{6.6}{7.2}\right)^2 \times \left(\frac{100}{85}\right)$$

$$= 0.0988 \text{ p.u.}$$

<u>REACTANCE OF TL</u>:

$$X_{p.u.} = \frac{X \times MVA_b}{KV_b^2}$$

$$= \frac{20 \times 100}{(120)^2} = 0.139 \text{ p.u.}$$

<u>REACTANCE OF T_2</u>:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.08 \times \left(\frac{110}{120}\right)^2 \times \left(\frac{100}{90}\right)$$

$$= 0.0747 \text{ p.u.}$$

REACTANCE OF M1:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.12 \times \left(\frac{6.2}{7.2}\right)^2 \times \left(\frac{100}{35}\right)$$

$$= 0.254 \text{ p.u.}$$

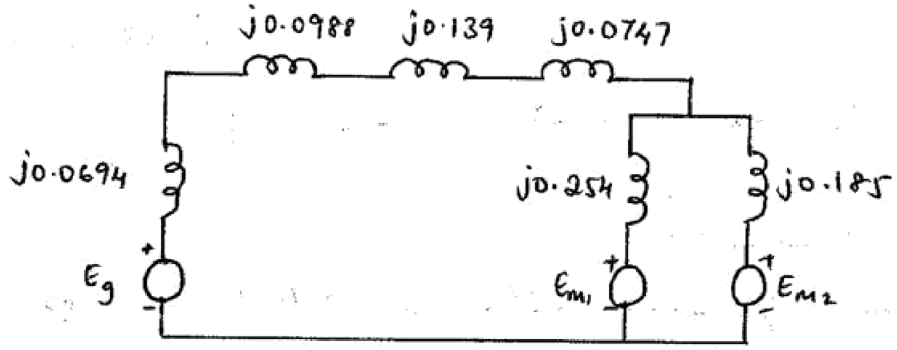
REACTANCE OF M2:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

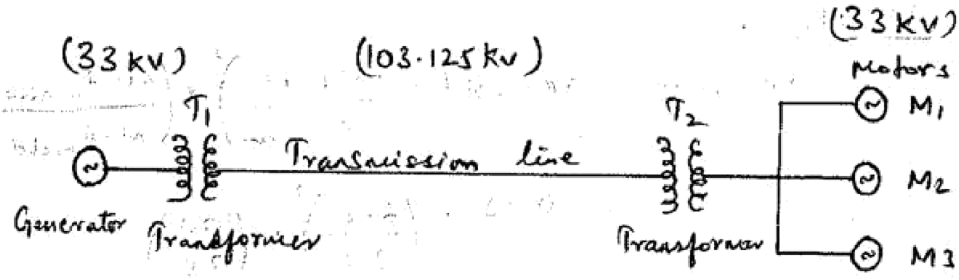
$$= 0.1 \times \left(\frac{6.2}{7.2}\right)^2 \times \left(\frac{100}{40}\right)$$

$$= 0.185 \text{ p.u.}$$

REACTANCE DIAGRAM:



10. A 100MVA, 33KV, 3Φ generator has a subtransient reactance of 15%. The generator is connected to motors through a transmission lines & transformers as shown. The motors have rated inputs of 30MVA, 20MVA and 50MVA at 30KV with 20% subtransient reactance. The 3Φ transformers are rated at 11MVA, 32KV Δ/100KV Y with the leakage reactance 8%. The line has a reactance of 50Ω. Selecting the generator rating as the base quantities in the generator circuit, determine the base quantities in the other part of the system. Evaluate the corresponding p.u. values. Hence draw the reactance diagram



(DECEMBER 2011 - 12M)

Sol Let T_1 be a $\Delta - Y$ transformer
& T_2 be a $Y - \Delta$ transformer
Let motors be M_1, M_2 & M_3

$$MVA_b = 100 MVA$$

In generator ckt $KV_b = 33 KV$

In $50\,\Omega$ Transmission line $KV_b = 33 \times \frac{100}{32}$

$$= 103.125 KV$$

In motors circuit $KV_b = 103.125 \times \frac{32}{100} = 33 KV$

REACTANCE OF GENERATOR:

Since generator rating is selected as base values its p.u. reactance remains same

$$\therefore X_{p.u.,new} = 0.15$$

REACTANCE OF TRANSFORMERS T1 & T2:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.08 \times \left(\frac{32}{33}\right)^2 \times \left(\frac{100}{11}\right)$$

$$= 0.684 \; p.u.$$

REACTANCE OF TRANSMISSION LINE:

$$X_{p.u.} = \frac{X \times MVA_b}{KV_b^2}$$

$$= \frac{50 \times 100}{(103.125)^2} = 0.47 \; p.u.$$

Reactance Of Motor M1:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.2 \times \left(\frac{30}{33}\right)^2 \times \left(\frac{100}{30}\right)$$

$$= 0.551\,p.u.$$

Reactance Of Motor M2:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.2 \times \left(\frac{30}{33}\right) \times \left(\frac{100}{20}\right)$$

$$= 0.826\,p.u.$$

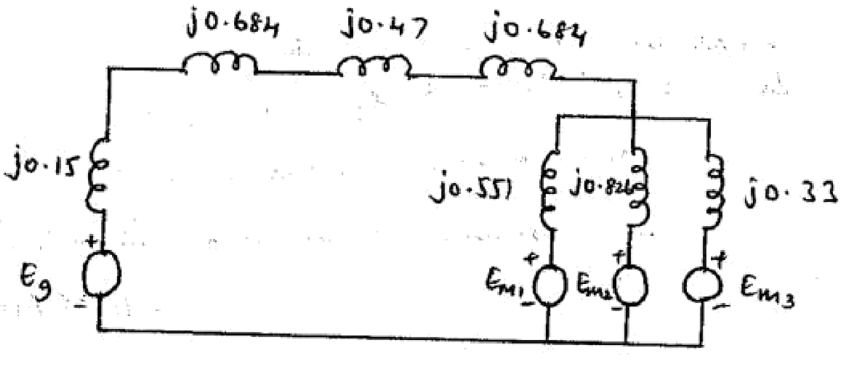
Reactance Of Motor M3:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.2 \times \left(\frac{30}{33}\right)^2 \times \left(\frac{100}{50}\right)$$

$$= 0.33\,p.u.$$

Reactance Diagram:



11. The one line diagram of unloaded ps is shown. Choose a base of 30MVA, 6.6 KV in G1 circuit & draw the p.u. circuit diagram.

G1 : 25MVA, 6.6KV, $X_{g1} = 0.2$ pu

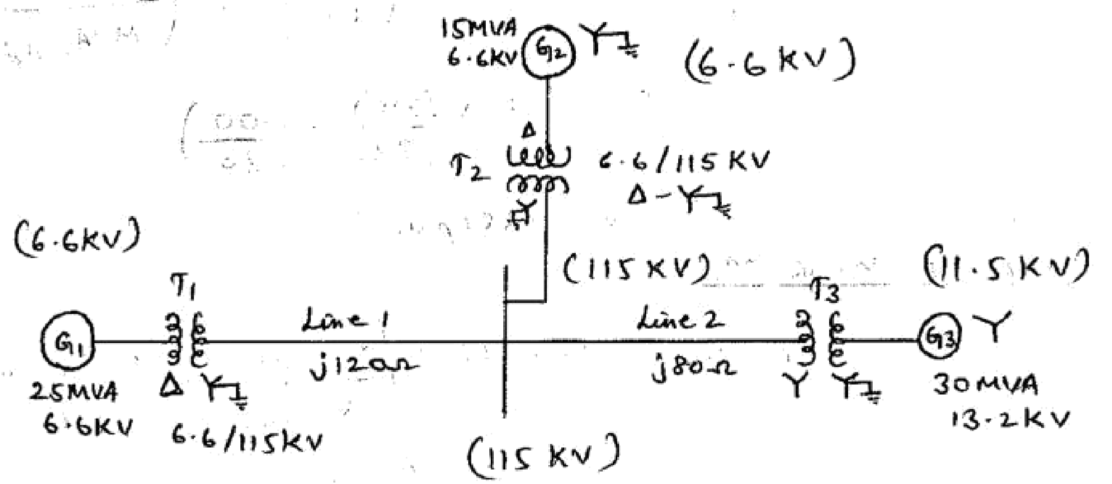
G2 : 15MVA, 6.6KV, $X_{g2} = 0.15$ pu

G3 = 30MVA, 13.2KV, $X_{g3} = 0.15$ pu

T1 = 30MVA, 6.6Δ – 115Y KV, $X_{T1} = 0.1$ pu

T_2 : 15MVA, 6.6Δ − 115Y KV, $x_{T_2} = 0.1$pu

T_3 : 3 single phase each rated 1MVA, 6.9/69 KV, $x_{T_3} = 0.1$ pu



[DECEMBER 2011 − 12M
(2002 SCHEME)]

Sol

$MVA_b = 30$ MVA

In G1 $KV_b = 6.6$ KV

In transmission line that includes line1, line2 as well as the line joining the star side of T_2

$$KV_b = 6.6 \times \frac{115}{6.6} = 115 \text{ KV}$$

In G2 $KV_b = 115 \times \frac{6.6}{115} = 6.6$ KV

Transformer T_3 is a bank of three 1ϕ transformers
∴ The 3ϕ rating of transformer T_3 is

MVA rating : 3 × 1MVA = 3MVA

line-to-line Voltage ratio : $\sqrt{3} \times 6.9 / \sqrt{3} \times 69$ KV
= 11.95/119.51 KV

In G3 $KV_b = 115 \times \frac{11.95}{119.51} = 11.5$ KV

REACTANCE OF G1:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.2 \times \left(\frac{6.6}{6.6}\right)^2 \times \left(\frac{30}{25}\right) = 0.24 \text{ p.u.}$$

REACTANCE OF T1:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.1 \times \left(\frac{6.6}{6.6}\right)^2 \times \left(\frac{30}{30}\right)$$

$$= 0.1 \ p.u.$$

REACTANCE OF T.L:

Line 1 : $X_{p.u.} = \dfrac{X \times MVA_b}{KV_b^2}$

$$= \frac{120 \times 30}{(115)^2}$$

$$= 0.272 \ p.u.$$

Line 2 : $X_{p.u.} = \dfrac{X \times MVA_b}{KV_b^2}$

$$= \frac{80 \times 30}{(115)^2}$$

$$= 0.181 \ p.u.$$

REACTANCE OF T2:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right) \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2$$

$$= 0.1 \times \left(\frac{30}{15}\right) \times \left(\frac{115}{115}\right)^2$$

$$= 0.2 \ p.u.$$

REACTANCE OF G2:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.15 \times \left(\frac{6.6}{6.6}\right)^2 \times \left(\frac{30}{15}\right)$$

$$= 0.3 \ p.u.$$

REACTANCE OF T3:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.1 \times \left(\frac{11.95}{11.5}\right)^2 \times \left(\frac{30}{3}\right)$$

$$= 1.08 \text{ p.u.}$$

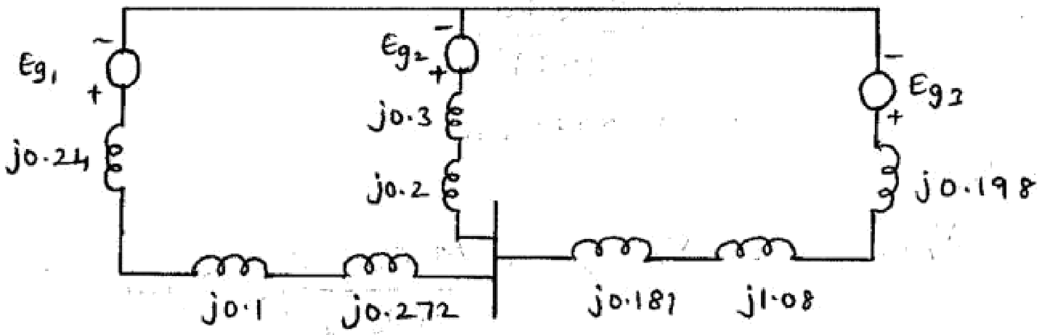
REACTANCE OF G3:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.15 \times \left(\frac{13.2}{11.5}\right)^2 \times \left(\frac{30}{30}\right)$$

$$= 0.198 \text{ p.u.}$$

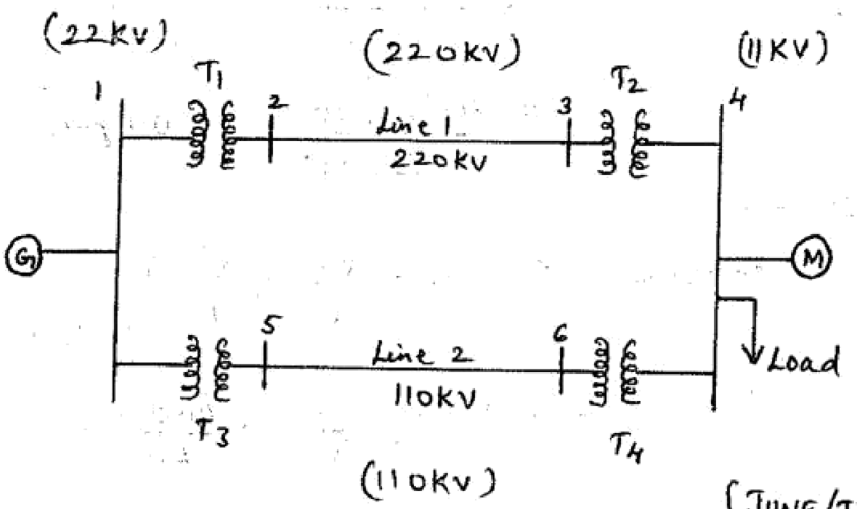
REACTANCE DIAGRAM:



12. The one line diagram of a 3ϕ PS is shown. Select a common base of 100MVA & 22KV on the generator side. Draw an impedance diagram with all impedances including the load impedance marked in p.u. The manufacturer's data for each device is given as follows:

G:	90MVA	22KV	$X = 18\%$
T1:	50MVA	22/220KV	$X = 10\%$
T2:	40MVA	220/11KV	$X = 6\%$
T3:	40MVA	22/110KV	$X = 6.4\%$
T4:	40MVA	110/11 KV	$X = 8\%$
M:	66.5MVA	10.45KV	$X = 18.5\%$

The 3ϕ load at bus 4 absorbs 57MVA, 0.6 pf lagging at 10.45KV. Line 1 & line 2 have reactances of 48.4Ω & 65.43Ω respectively.



[JUNE/JULY 2011 - 15M
2002 SCHEME]

Sol:

$MVA_b = 100MVA$

In G $KV_b = 22KV$

In Line 1 $KV_b = 22 \times \frac{220}{22} = 220KV$

In Line 2 $KV_b = 22 \times \frac{110}{22} = 110KV$

In M & Load $KV_b = 220 \times \frac{11}{220} = 11KV$

OR

$110 \times \frac{11}{110} = 11KV$

REACTANCE OF G:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.18 \times \left(\frac{22}{22}\right)^2 \times \left(\frac{100}{90}\right)$$

$$= 0.2 \text{ p.u.}$$

REACTANCE OF T_1:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.1 \times \left(\frac{22}{22}\right)^2 \times \left(\frac{100}{50}\right) = 0.2 \text{ p.u.}$$

REACTANCE OF LINE 1:

$$X_{p.u.} = \frac{X \times MVA_b}{KV_b^2}$$

$$= \frac{48.4 \times 100}{(220)^2} = 0.1 \text{ p.u.}$$

REACTANCE OF T2:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.06 \times \left(\frac{220}{220}\right)^2 \times \left(\frac{100}{40}\right) = 0.15 \text{ p.u.}$$

REACTANCE OF T3:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.064 \times \left(\frac{22}{22}\right)^2 \times \left(\frac{100}{40}\right) = 0.16 \text{ p.u.}$$

REACTANCE OF LINE 2:

$$X_{p.u.} = \frac{X \times MVA_b}{KV_b^2} = \frac{65.43 \times 100}{(110)^2} = 0.54 \text{ p.u.}$$

REACTANCE OF T4:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.08 \times \left(\frac{110}{110}\right)^2 \times \left(\frac{100}{40}\right) = 0.2 \text{ p.u.}$$

REACTANCE OF M:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.185 \times \left(\frac{10.45}{11}\right)^2 \times \left(\frac{100}{66.5}\right)$$

$$= 0.251 \text{ p.u.}$$

IMPEDANCE OF LOAD:

Given that Load has

Magnitude of 3ϕ Complex power, $|S| = \sqrt{P^2 + Q^2} = 57 \text{ MVA}$

Lagging power factor, $\cos\phi = 0.6$

$\phi = \cos^{-1}(0.6) = 53.13°$

& magnitude of line Voltage, $|V_L| = 10.45 \text{ kV}$

3ϕ Complex power, $S = P + jQ = |S| \angle \phi$

$$= 57 \angle 53.13 \text{ MVA}$$

Now, $S^* = P - jQ = |S| \angle -\phi$

$$= 57 \angle -53.13 \text{ MVA}$$

W.K.T 3ϕ load impedance Z_L can be calculated as

$$Z_L = \frac{|V_L|^2}{P - jQ} \quad \text{(Assuming a Star Connected load)}$$

$$= \frac{(10.45)^2}{57 \angle -53.13} = 1.916 \angle 53.13 \ \Omega$$

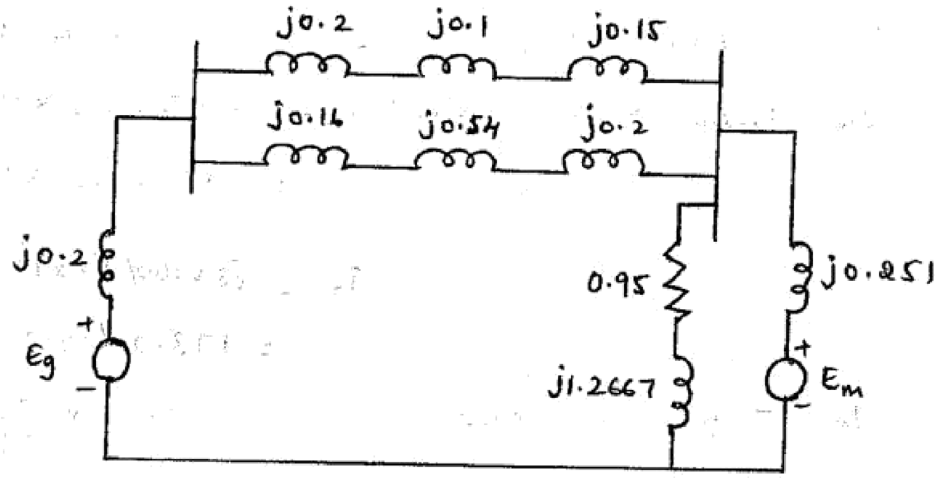
$$= (1.1496 + j1.5328)\ \Omega$$

$$\therefore Z_{L(p.u.)} = \frac{Z_L \times MVA_b}{KV_b^2}$$

$$= \frac{(1.1496 + j1.5328) \times 100}{(11)^2}$$

$$= 0.95 + j1.2667 \text{ p.u.}$$

IMPEDANCE DIAGRAM:

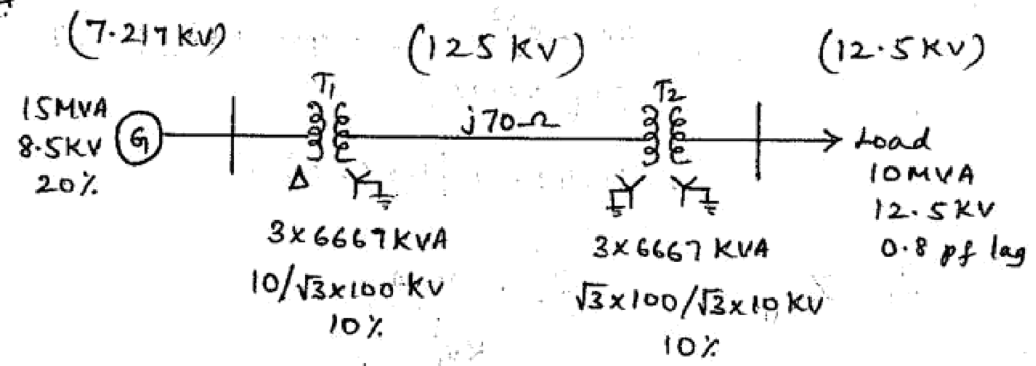


13. A 15MVA, 8.5KV, 3ϕ generator has a subtransient reactance of 20%. It is connected through a Δ-Y transformer to a HV TL having a total series reactance of 70Ω. The load end of the line

has Y-Y stepdown transformer. Both transformer banks are composed of 1ϕ transformers connected for 3ϕ operation. Each of three transformers composing 3ϕ bank is rated 6667 KVA, 10/100 KV, with a reactance of 10%. The load represented as impedance, is drawing 10 MVA at 12.5 KV and 0.8 pf lagging. Draw the SLD of the power network. choose a base of 10 MVA, 12.5 KV in the load circuit and determine the reactance diagram. Determine also the voltage at the terminals of the generator

(JUNE-JULY 2009 -12M)

Sol



$$MVA_b = 10\ MVA$$

In Load circuit $KV_b = 12.5\ KV$

Transformer banks T_1 & T_2 are composed of three 1ϕ transformers each.

$\therefore$ The 3ϕ ratings of T_1 & T_2 are

MVA rating : $3 \times 6667\ kVA = 20\ MVA$

Line-to-Line Voltage ratio : $T_1 = 10/\sqrt{3}\times100\ KV$

$= 10/173.2\ KV$

$T_2 = \sqrt{3}\times100/\sqrt{3}\times10\ KV$

$= 173.2/17.32\ KV$

In 70Ω transmission line $KV_b = 12.5 \times \dfrac{173.2}{17.32}$

$= 125\ KV$

In G $KV_b = 125 \times \dfrac{10}{173.2} = 7.217\ KV$

<u>REACTANCE OF G:</u>

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.2 \times \left(\frac{8.5}{7.217}\right)^2 \times \left(\frac{10}{15}\right)$$

$$= 0.185 \text{ p.u.}$$

<u>REACTANCE OF TRANSFORMERS T1:</u>

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.1 \times \left(\frac{10}{7.217}\right)^2 \times \left(\frac{10}{20}\right) = 0.096 \text{ p.u.}$$

<u>REACTANCE OF T.L:</u>

$$X_{p.u.} = \frac{X \times MVA_b}{KV_b^2} = \frac{70 \times 10}{(125)^2} = 0.0448 \text{ p.u.}$$

<u>REACTANCE OF T2:</u>

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.1 \times \left(\frac{173.2}{125}\right)^2 \times \left(\frac{10}{20}\right) = 0.096 \text{ p.u.}$$

<u>IMPEDANCE OF LOAD:</u>

$$|S| = 10 \text{ MVA}$$

p.f lag, $\cos\phi = 0.8$

$$\phi = \cos^{-1}(0.8) = 36.87^\circ$$

$\therefore$ 3ϕ Complex power $S = 10\angle 36.87$ MVA

$$S^* = P - jQ = 10\angle -36.87 \text{ MVA}$$

Magnitude of line voltage $|V_L| = 12.5$ KV

3ϕ Load impedance, $Z_L = \frac{|V_L|^2}{P-jQ} = \frac{(12.5)^2}{10\angle -36.87}$

$$= 15.625\angle 36.87 \ \Omega$$

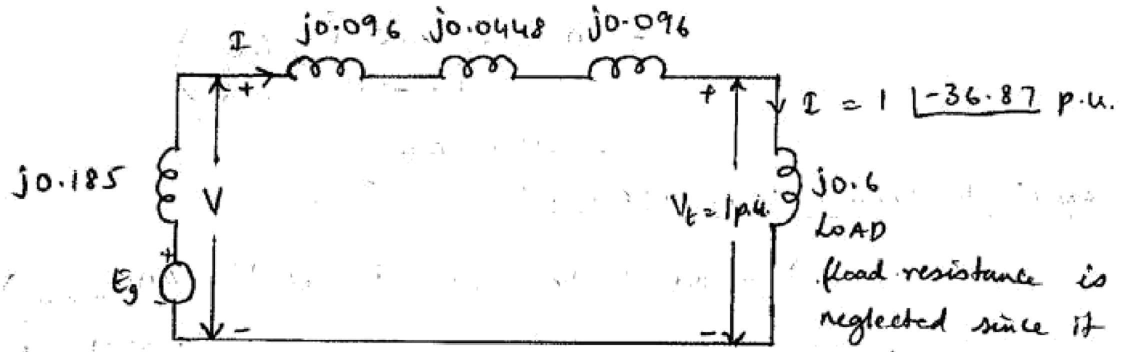
$$= (12.5 + j9.375)\ \Omega$$

$\therefore$ $Z_{L(p.u.)} = \frac{Z_L \times MVA_b}{KV_b^2} = \frac{(12.5 + j9.375) \times 10}{(12.5)^2}$

$= 0.8 + j0.6$ p.u.

$= 1\angle 36.87$ p.u.

REACTANCE DIAGRAM:



(load resistance is neglected since it is a reactance diagram)

GENERATOR TERMINAL VOLTAGE:

Load voltage in p.u. $V_t = \dfrac{\text{Actual load voltage}}{\text{Base voltage}}$

$= \dfrac{12.5\text{ kV}}{12.5\text{ kV}} = 1$ p.u.

Load current in p.u. $I = \dfrac{V_t}{Z_{L(pu)}} = \dfrac{1}{1\angle 36.87}$

$= 1\angle -36.87$ p.u

Applying KVL to the reactance diagram in clockwise direction

$$V - j0.096\,I - j0.0448\,I - j0.096\,I - V_t = 0$$

$$V = V_t + I\,(j0.096 + j0.0448 + j0.096)$$

$$= 1 + 1\angle -36.87\,(j0.2368)$$

$$= 1 + 1\angle -36.87 \times 0.2368\angle 90$$

$$= 1 + 0.2368\angle 53.13$$

$$= 1 + 0.1421 + j0.1894$$

$$= 1.1421 + j0.1894$$

$$= 1.1577\angle 9.41 \text{ p.u.}$$

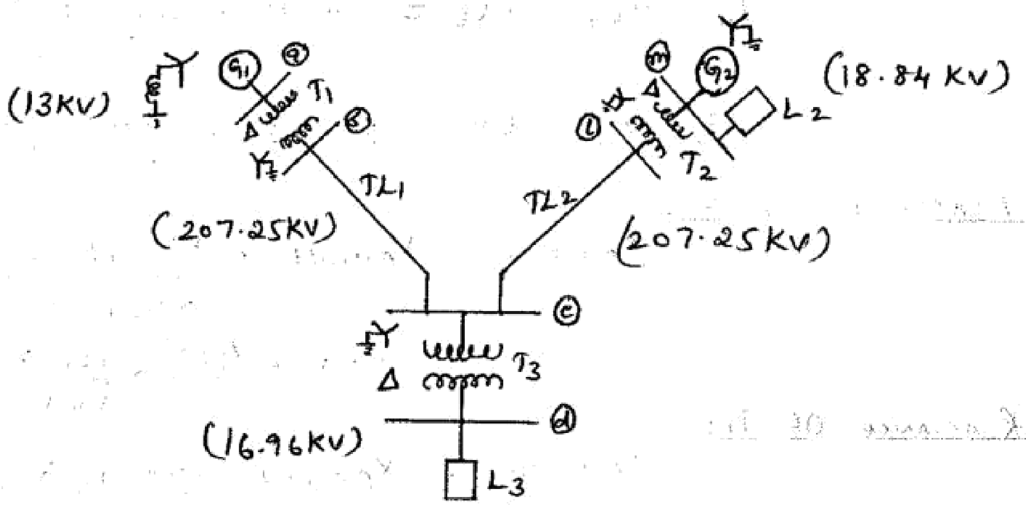
Actual value of V = p.u. value of V × Base KV in G

$$= 1.1577\angle 19.41 \times 7.217$$

$$= 8.355\angle 19.41 \text{ KV}$$

$$= 8.24 + j1.36$$

14. Draw the impedance diagram for the PS shown. Mark on it the per unit impedances calculated on a base of 50 MVA, 13 KV in the circuit of generator G_1



G1 : 25 MVA, 13 KV, $X_d'' = 0.15$ p.u.

T1 : 30 MVA, 220Y/13.8Δ KV, $X = 10\%$

TL_1 : $j60\ \Omega$

T_3 : Bank of 1ø transformers, each rated 10MVA, 127/18 KV, $X = 8\%$

L_3 : $4 + j2\ \Omega$

TL_2 : $j90\ \Omega$

T_2 : 40 MVA, 220/20 KV, $X = 12\%$

G_2 : 35 MVA, 22 KV, $X_d'' = 0.12$ p.u.

L_2 : $3 + j1\ \Omega$

(JULY/AUGUST 2005 - 14M)

Sol

$MVA_b = 50$ MVA

In G_1 $KV_b = 13$ KV

In TL_1 $KV_b = 13 \times \frac{220}{13.8} = 207.25$ KV

T_3 is a bank of three 1ϕ transformers hence its 3ϕ rating is

MVA rating: $3 \times 10\,MVA$
$= 30\,MVA$

Line-to-line voltage ratio: $\sqrt{3} \times 127/18\ kV$
$= 220/18\,kV$

In L_3 $\quad KV_b = 207.25 \times \dfrac{18}{220} = 16.96\ KV$

In TL_2 $\quad KV_b = 16.96 \times \dfrac{220}{18} = 207.25\ KV$

In G_2 $\quad KV_b = 207.25 \times \dfrac{20}{220} = 18.84\ KV$

REACTANCE OF G1:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.15 \times \left(\frac{13}{13}\right)^2 \times \left(\frac{50}{25}\right) = 0.3\ p.u.$$

REACTANCE OF T1:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.1 \times \left(\frac{13.8}{13}\right)^2 \times \left(\frac{50}{30}\right) = 0.188\ p.u.$$

REACTANCE OF TL1:

$$X_{p.u.} = \frac{X \times MVA_b}{KV_b^2} = \frac{60 \times 50}{(207.25)^2} = 0.0698\ p.u.$$

REACTANCE OF T3:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.08 \times \left(\frac{220}{207.25}\right)^2 \times \left(\frac{50}{30}\right) = 0.15\ p.u.$$

IMPEDANCE OF L3:

$$Z_{L3(p.u.)} = \frac{Z_{L3} \times MVA_b}{KV_b^2} = \frac{(4+j2) \times 50}{(16.96)^2} = 0.695 + j0.347\ p.u.$$

REACTANCE OF TL2:

$$X_{p.u.} = \frac{X \times MVA_b}{KV_b^2} = \frac{90 \times 50}{(207.25)^2} = 0.104\ p.u.$$

REACTANCE OF T2:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.12 \times \left(\frac{220}{207.25}\right)^2 \times \left(\frac{50}{40}\right) = 0.169\ p.u.$$

REACTANCE OF G2:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

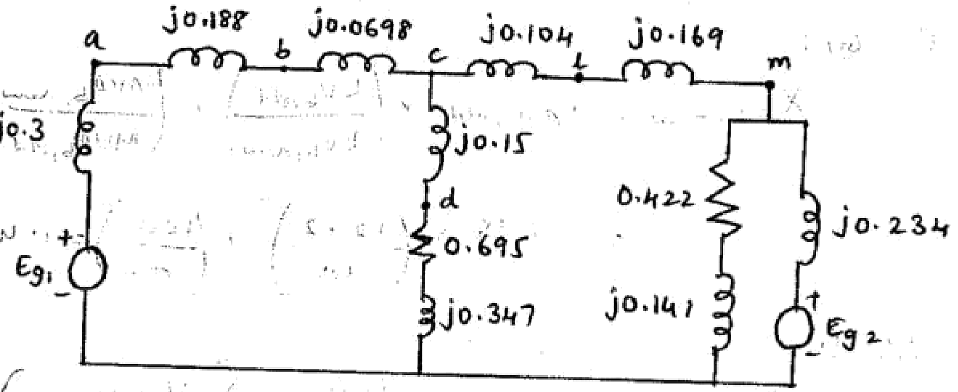
$$= 0.12 \times \left(\frac{22}{18.84}\right)^2 \times \left(\frac{50}{35}\right) = 0.234 \text{ p.u.}$$

IMPEDANCE OF L2:

$$Z_{L2(p.u.)} = \frac{Z_{L2} \times MVA_b}{KV_b^2} = \frac{(3+j1) \times 50}{(18.84)^2}$$

$$= 0.422 + j0.141 \text{ p.u.}$$

IMPEDANCE DIAGRAM:



15. Prepare the per phase schematic of the system shown. Show all impedances in p.u. on a 100MVA, 132KV base in the T.L circuit. Given

G1 : 50MVA, 12.2KV, X = 0.15p.u.

G2 : 20MVA, 13.8KV, X = 0.15p.u.

T1 : 80MVA, 12.2/161 KV, X = 0.1p.u.

T2 : 40MVA, 13.8/161 KV, X = 0.1p.u.

Load : 50MVA, 0.8 p.f lagging, operating at 154KV

Determine the p.u. impedance of the load for the following cases:

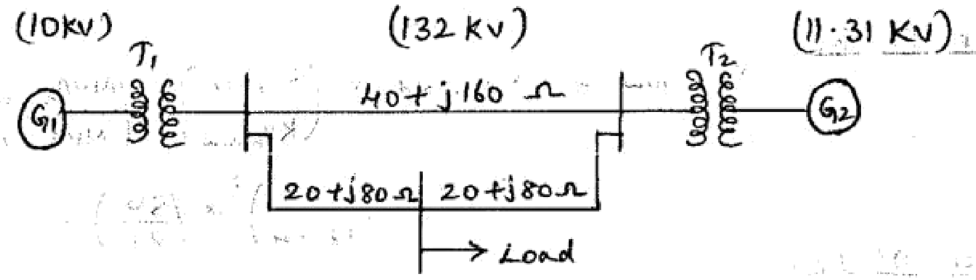
a. Load modelled as the series combination of resistance and inductance

b. Load modelled as the parallel combination of resistance & inductance

Sol

$MVA_b = 100MVA$ (Figure is shown in the next page)

In T.L $KV_b = 132KV$



$$\text{In } G1 \quad KV_b = 132 \times \frac{12.2}{161} = 10\ KV$$

$$\text{In } G2 \quad KV_b = 132 \times \frac{13.8}{161} = 11.31\ KV$$

Reactance of G1:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.15 \times \left(\frac{12.2}{10}\right)^2 \times \left(\frac{100}{50}\right) = 0.4465\ p.u.$$

Reactance of T1:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.1 \times \left(\frac{12.2}{10}\right)^2 \times \left(\frac{100}{80}\right) = 0.186\ p.u.$$

Impedance of T-L:

(i) 40+j160 Ω T.L:

$$Z_{p.u.} = \frac{Z \times MVA_b}{KV_b^2} = \frac{(40+j160) \times 100}{(132)^2}$$

$$= 0.2296 + j0.9183\ p.u.$$

(ii) 20+j80 Ω T.L:

$$Z_{p.u.} = \frac{Z \times MVA_b}{KV_b^2} = \frac{(20+j80) \times 100}{(132)^2}$$

$$= 0.1148 + j0.4591\ p.u.$$

Reactance of T2:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.1 \times \left(\frac{161}{132}\right)^2 \times \left(\frac{100}{40}\right) = 0.3719\ p.u.$$

Reactance of G2:

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.15 \times \left(\frac{13.8}{11.31}\right)^2 \times \left(\frac{100}{20}\right) = 1.1166\ p.u.$$

IMPEDANCE OF LOAD:

$$|S| = 50\,MVA$$

Lagging pf $\cos\phi = 0.8$

$$\phi = \cos^{-1}(0.8) = 36.87°$$

$\therefore$ 3ϕ complex power $S = |S| \angle \phi = 50 \angle 36.87$ MVA

$$S = P + jQ = 40 + j30 \text{ MVA}$$

Conjugate of 3ϕ Complex power $S^* = |S| \angle -\phi = 50 \angle -36.87$ MVA

$$S^* = P - jQ = 40 - j30 \text{ MVA}$$

Magnitude of line voltage $|V_L| = 154$ KV

a. SERIES COMBINATION OF RESISTANCE AND INDUCTANCE

$$Z_L = \frac{|V_L|^2}{P - jQ} = \frac{(154)^2}{40 - j30} = (379.456 + j284.592)\,\Omega$$

$$Z_{L(p.u.)} = \frac{Z_L \times MVA_b}{KV_b^2} = \frac{(379.456 + j284.592) \times 100}{(132)^2}$$

$$= 2.18 + j1.63 \text{ p.u.}$$

b. PARALLEL COMBINATION OF RESISTANCE AND INDUCTANCE:

3ϕ Complex power $S = P + jQ = 40 + j30$ MVA

$$\therefore P = 40\,MW$$
$$Q = 30\,MVAr$$

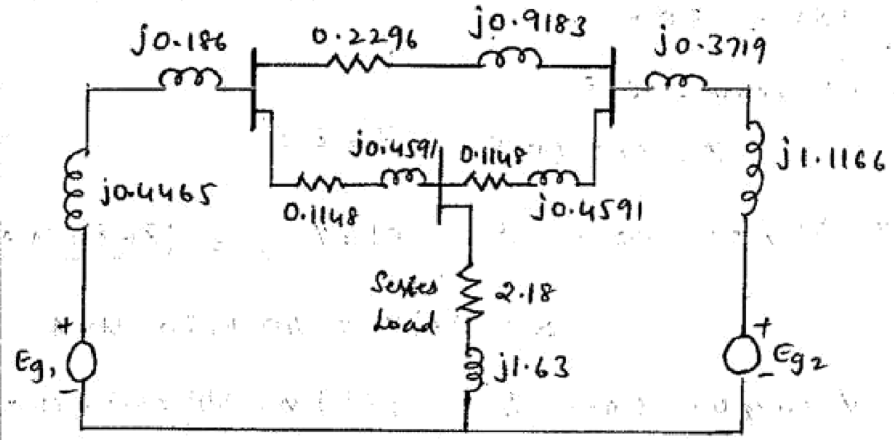
$$R_L = \frac{|V_L|^2}{P} = \frac{(154)^2}{40} = 592.9\,\Omega$$

$$X_L = \frac{|V_L|^2}{Q} = \frac{(154)^2}{30} = 790.53\,\Omega$$

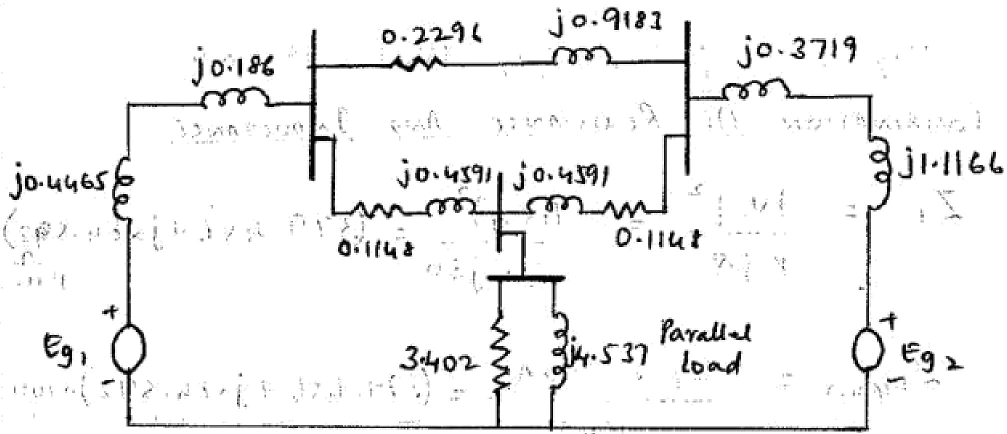
$$R_{Lp.u.} = \frac{R_L \times MVA_b}{KV_b^2} = \frac{592.9 \times 100}{(132)^2} = 3.402 \text{ p.u.}$$

$$X_{Lp.u.} = \frac{X_L \times MVA_b}{KV_b^2} = \frac{790.53 \times 100}{(132)^2} = 4.537 \text{ p.u.}$$

IMPEDANCE DIAGRAM:

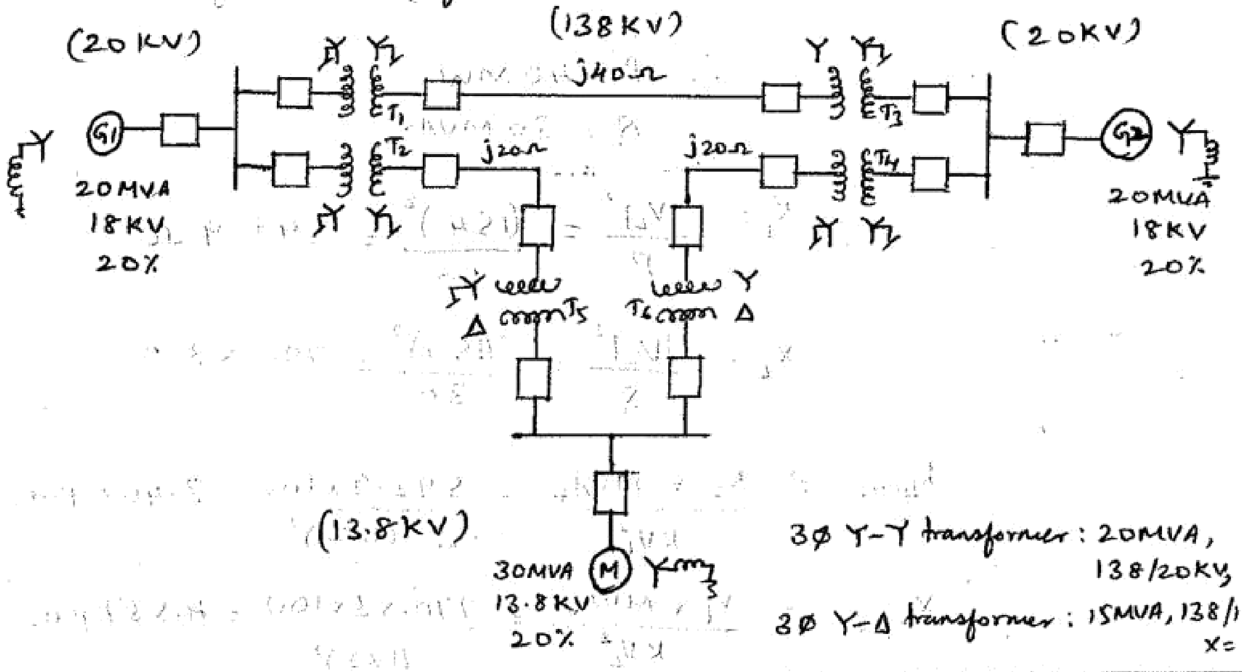


(a)



(b)

16. Draw the reactance diagram for the PS shown. The rating of generator, motor & transformers are given below. Neglect resistance & use a base of 50MVA, 138KV in the 40Ω line



3φ Y-Y transformer: 20MVA, 138/20KV, 10%

3φ Y-Δ transformer: 15MVA, 138/13.8KV, X=10%

Sol

$MVA_b = 50\,MVA$

In 40Ω T.L $KV_b = 138\,KV$

In G_1 $KV_b = 138 \times \frac{20}{138} = 20\,KV$

In G_2 $KV_b = 138 \times \frac{20}{138} = 20\,KV$

In M $KV_b = 138 \times \frac{13.8}{138} = 13.8\,KV$

<u>REACTANCE OF G1 AND G2:</u>

Since G_1 & G_2 have same rating their p.u. reactance will also be same

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.2 \times \left(\frac{18}{20}\right)^2 \times \left(\frac{50}{20}\right) = 0.405\,p.u.$$

<u>REACTANCE OF T_1, T_2, T_3 AND T_4:</u>

Since T_1, T_2, T_3 & T_4 are identical transformers their p.u. reactance remains same

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.1 \times \left(\frac{20}{20}\right)^2 \times \left(\frac{50}{20}\right) = 0.25\,p.u.$$

<u>REACTANCE OF T.L:</u>

(i) <u>$j40\Omega$ T.L:</u>

$$X_{p.u.} = \frac{X \times MVA_L}{KV_L^2} = \frac{40 \times 50}{(138)^2} = 0.105\,p.u.$$

(ii) <u>$j20\Omega$ T.L:</u>

$$X_{p.u.} = \frac{X \times MVA_b}{KV_L} = \frac{20 \times 50}{(138)^2} = 0.0525\,p.u.$$

<u>REACTANCE OF T_5 AND T_6:</u>

T_5 & T_6 are identical transformers hence their p.u. reactance is same

$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.1 \times \left(\frac{138}{138}\right)^2 \times \left(\frac{50}{15}\right)$$

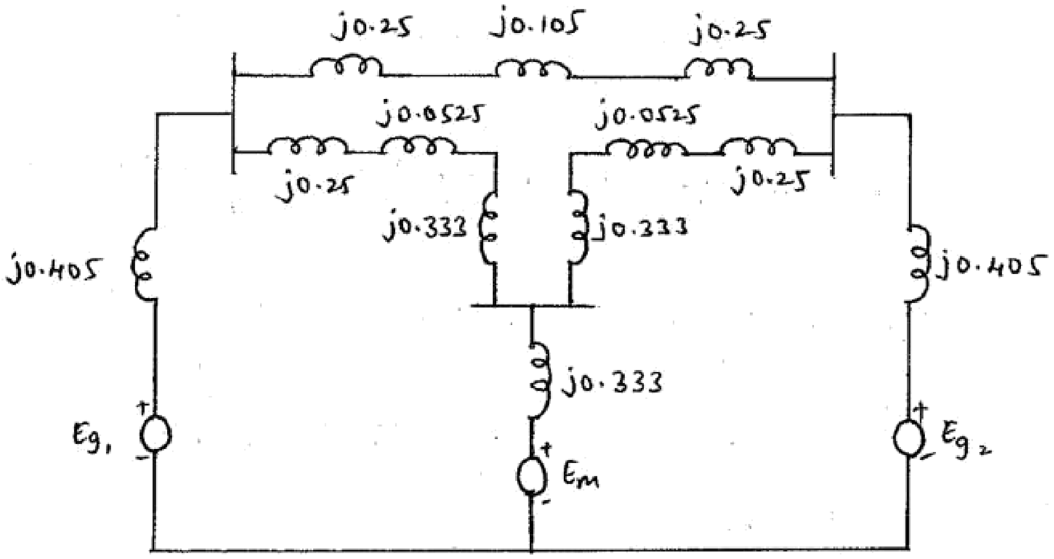
$$= 0.333\,p.u.$$

REACTANCE OF M:

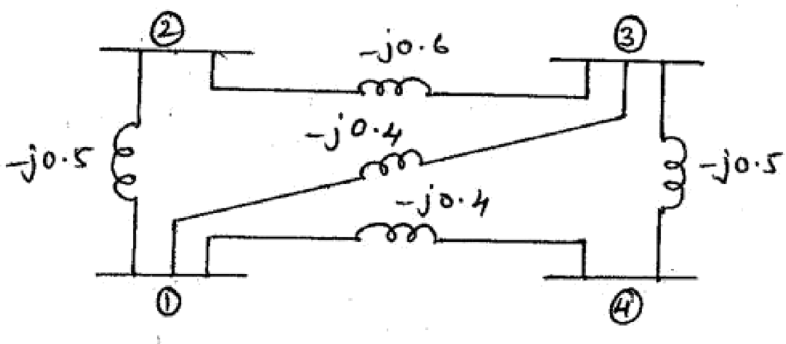
$$X_{p.u.,new} = X_{p.u.,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.2 \times \left(\frac{13.8}{13.8}\right)^2 \times \left(\frac{50}{30}\right) = 0.333 \text{ p.u.}$$

REACTANCE DIAGRAM:



17. For the network shown form the admittance matrix. The values are marked in p.u.



(JUNE-JULY 2009-8M)

Sol: There are 4 major nodes in the network shown. By using inspection method the self & mutual admittances can be written as

$$Y_{11} = -j0.5 - j0.4 - j0.4 = -j1.3 \quad \left(\because Y_{ii} = \sum_{j=0}^{n} y_{ij} \quad i \neq j\right)$$

$$Y_{22} = -j0.5 - j0.6 = -j1.1$$

$$Y_{33} = -j0.4 - j0.6 - j0.5 = -j1.5$$

$$Y_{44} = -j0.4 - j0.5 = -j0.9$$

$$Y_{12} = Y_{21} = -(-j0.5) = j0.5 \quad (\because Y_{ij} = Y_{ji} = -y_{ij})$$

$$Y_{13} = Y_{31} = -(-j0.4) = j0.4$$

$Y_{14} = Y_{41} = -(-j0.4) = j0.4$

$Y_{23} = Y_{32} = -(-j0.6) = j0.6$

$Y_{24} = Y_{42} = 0$ ($\because$ There is no connection b/w nodes 2 & 4)

$Y_{34} = Y_{43} = -(-j0.5) = j0.5$

$\therefore$ The bus admittance matrix Y_{bus} is

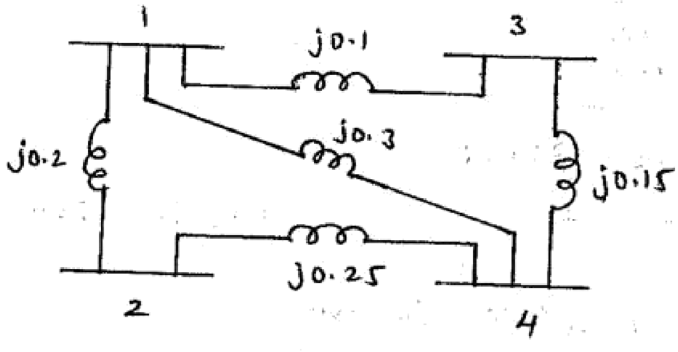
$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} = \begin{bmatrix} -j1.3 & j0.5 & j0.4 & j0.4 \\ j0.5 & -j1.1 & j0.6 & 0 \\ j0.4 & j0.6 & -j1.5 & j0.5 \\ j0.4 & 0 & j0.5 & -j0.9 \end{bmatrix}$$

18. Determine Y_{BUS} by inspection method for the system details given in table.

Bus Code	1-2	2-4	3-4	3-1	1-4
Series reactance of the line	j0.2	j0.25	j0.15	j0.1	j0.3

(MAY-JUNE 2010 - 6M)

Sol: From the system details the system network is as shown



Self & mutual admittances are

$Y_{11} = \frac{1}{j0.2} + \frac{1}{j0.1} + \frac{1}{j0.3}$ (Reciprocal is taken because given values are the reactances)

$= -j18.33$

$Y_{22} = \frac{1}{j0.2} + \frac{1}{j0.25} = -j9$

$$Y_{33} = \frac{1}{j0.1} + \frac{1}{j0.15} = -j16.67$$

$$Y_{44} = \frac{1}{j0.3} + \frac{1}{j0.25} + \frac{1}{j0.15} = -j14$$

$$Y_{12} = Y_{21} = -\left(\frac{1}{j0.2}\right) = j5$$

$$Y_{13} = Y_{31} = -\left(\frac{1}{j0.1}\right) = j10$$

$$Y_{14} = Y_{41} = -\left(\frac{1}{j0.3}\right) = j3.33$$

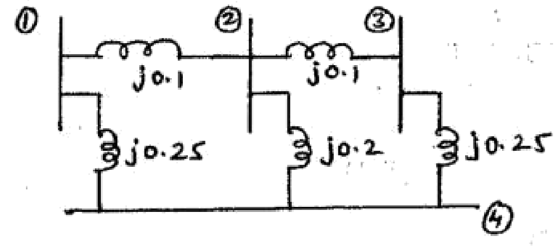
$$Y_{23} = Y_{32} = 0$$

$$Y_{24} = Y_{42} = -\left(\frac{1}{j0.25}\right) = j4$$

$$Y_{34} = Y_{43} = -\left(\frac{1}{j0.15}\right) = j6.67$$

$$\therefore\ Y_{BUS} = \begin{bmatrix} -j18.33 & j5 & j10 & j3.33 \\ j5 & -j9 & 0 & j4 \\ j10 & 0 & -j16.67 & j6.67 \\ j3.33 & j4 & j6.67 & -j14 \end{bmatrix}$$

19. For a four bus system shown determine bus admittance matrix. The pu line reactances are indicated on the diagram. Treat bus ④ as reference bus.



(JUNE-JULY 2011 - 5M)

Sol Self & mutual admittances are

$$Y_{11} = \frac{1}{j0.1} + \frac{1}{j0.25} = -j14$$

$$Y_{22} = \frac{1}{j0.1} + \frac{1}{j0.1} + \frac{1}{j0.2} = -j25$$

$$Y_{33} = \frac{1}{j0.1} + \frac{1}{j0.25} = -j14$$

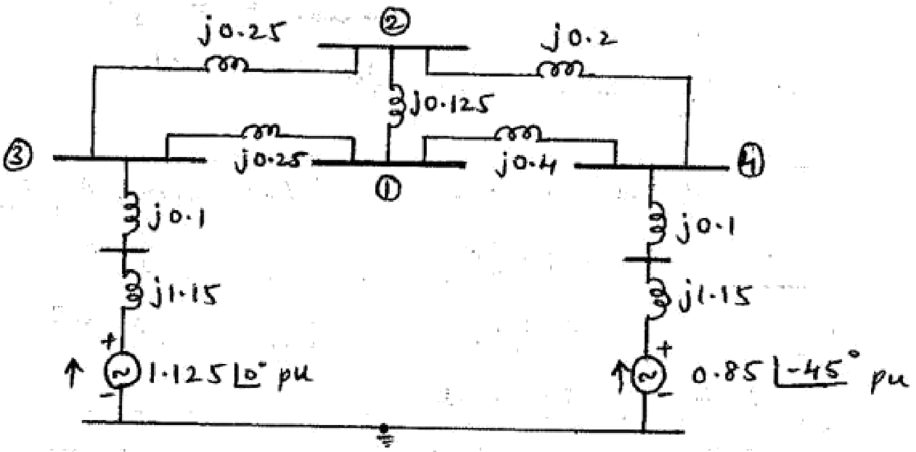
$$Y_{12} = Y_{21} = -\left(\frac{1}{j0.1}\right) = j10$$

$$Y_{13} = Y_{31} = 0$$

$$Y_{23} = Y_{32} = -\left(\frac{1}{j0.1}\right) = j10$$

$$\therefore Y_{bus} = \begin{bmatrix} -j14 & j10 & 0 \\ j10 & -j25 & j10 \\ 0 & j10 & -j14 \end{bmatrix}$$

20. Develop the bus admittance matrix for the system shown. All line reactance are marked in p.u.



(JUNE 2012 – 10M)

Sol

Self & mutual admittances are

$$Y_{11} = \frac{1}{j0.125} + \frac{1}{j0.25} + \frac{1}{j0.4} = -j14.5$$

$$Y_{22} = \frac{1}{j0.125} + \frac{1}{j0.25} + \frac{1}{j0.2} = -j17$$

$$Y_{33} = \frac{1}{j0.25} + \frac{1}{j0.1 + j1.15} + \frac{1}{j0.25}$$ (∵ j0.1 & j1.15 are in series)

$$= -j8.8$$

$$Y_{44} = \frac{1}{j0.1 + j1.15} + \frac{1}{j0.4} + \frac{1}{j0.2} = -j8.3$$

$$Y_{12} = Y_{21} = -\left(\frac{1}{j0.125}\right) = j8 \qquad Y_{34} = Y_{43} = 0$$

$$Y_{13} = Y_{31} = -\left(\frac{1}{j0.25}\right) = j4$$

$$Y_{14} = Y_{41} = -\left(\frac{1}{j0.4}\right) = j2.5$$

$$Y_{23} = Y_{32} = -\left(\frac{1}{j0.25}\right) = j4$$

$$Y_{24} = Y_{42} = -\left(\frac{1}{j0.2}\right) = j5$$

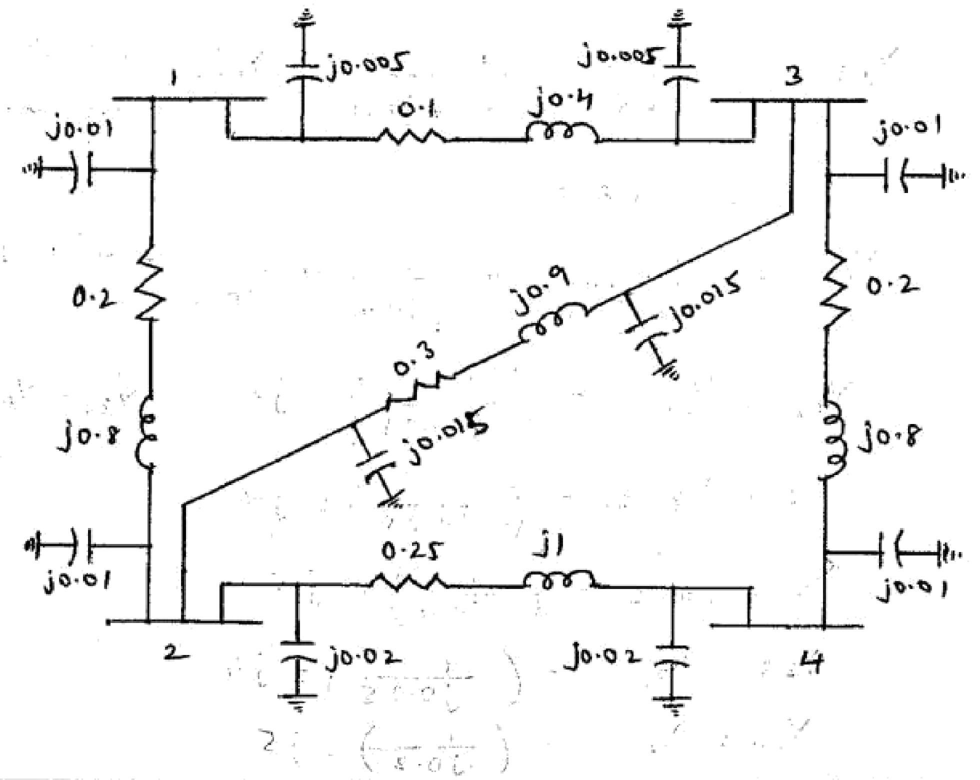
$$\therefore \; Y_{bus} = \begin{bmatrix} -j14.5 & j8 & j4 & j2.5 \\ j8 & -j17 & j4 & j5 \\ j4 & j4 & -j8.8 & 0 \\ j2.5 & j5 & 0 & -j8.3 \end{bmatrix}$$

21. Following data gives the series impedance and line charging admittance in p.u. on a common base for each line of a four bus PS. Obtain Y_{Bus} for the system

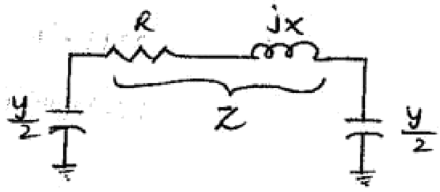
Bus Code	Line Impedance (pu)	Line Charging Admittance P.U.
1 − 2	0.2 + j0.8	j0.02
2 − 3	0.3 + j0.9	j0.03
2 − 4	0.25 + j1	j0.04
3 − 4	0.2 + j0.8	j0.02
1 − 3	0.1 + j0.4	j0.01

(JUNE - JULY 2008 - 12M)

Sol: From the data given the PS network with series impedance and line charging admittance (shunt admittance) can be drawn as shown



NOTE: If y is the shunt admittance (line charging admittance) of the line, then it is divided as $y/2$ at both ends of the line as shown.



Self and Mutual admittances are

$$Y_{11} = \frac{1}{0.2+j0.8} + \frac{1}{0.1+j0.4} + j0.005 + j0.01$$

$$= 0.8823 - j3.5144$$

$$Y_{22} = \frac{1}{j0.8+0.2} + \frac{1}{0.3+j0.9} + \frac{1}{0.25+j1} + j0.01 + j0.015 + j0.02$$

$$= 0.8627 - j3.0726$$

$$Y_{33} = \frac{1}{0.1+j0.4} + \frac{1}{0.3+j0.9} + \frac{1}{0.2+j0.8} + j0.005 + j0.015 + j0.01$$

$$= 1.2157 - j4.4994$$

$$Y_{44} = \frac{1}{0.25+j1} + \frac{1}{0.2+j0.8} + j0.02 + j0.01$$

$$= 0.5294 - j2.0876$$

$$Y_{12} = Y_{21} = -\left(\frac{1}{0.2+j0.8}\right) = -0.2941 + j1.1764$$

$$Y_{13} = Y_{31} = -\left(\frac{1}{0.1+j0.4}\right) = -0.5882 + j2.353$$

$$Y_{14} = Y_{41} = 0$$

$$Y_{23} = Y_{32} = -\left(\frac{1}{0.3+j0.9}\right) = -0.3333 + j1$$

$$Y_{24} = Y_{42} = -\left(\frac{1}{0.25+j1}\right) = -0.2353 + j0.9411$$

$$Y_{34} = Y_{43} = -\left(\frac{1}{0.2+j0.8}\right) = -0.2941 + j1.1764$$

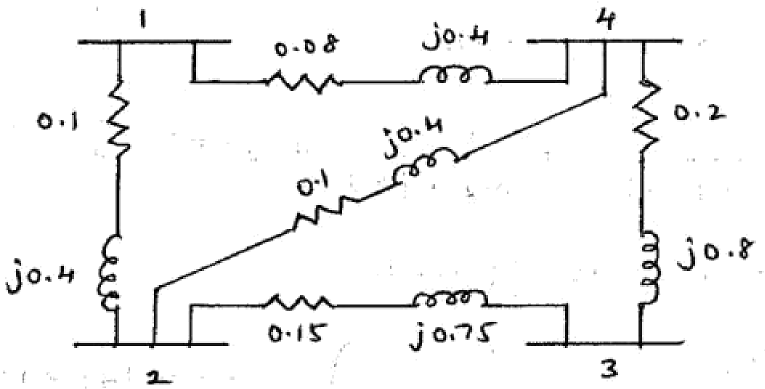
$$\therefore Y_{BUS} = \begin{bmatrix} 0.8823-j3.5144 & -0.2941+j1.1764 & -0.5882+j2.353 & 0 \\ -0.2941+j1.1764 & 0.8627-j3.0726 & -0.3333+j1 & -0.2353+j0.9411 \\ -0.5882+j2.353 & -0.3333+j1 & 1.2157-j4.4994 & -0.2941+j1.1764 \\ 0 & -0.2353+j0.9411 & -0.2941+j1.1764 & 0.5294-j2.0876 \end{bmatrix}$$

22. Following data gives the series impedances in per unit on a common base for each transmission line of a four bus PS. Obtain the Y_{BUS} for the system

Sl. No.	Transmission line between buses	Series Impedance
1	1 - 2	0.1 + j0.4
2	2 - 3	0.15 + j0.75
3	3 - 4	0.2 + j0.8
4	4 - 1	0.08 + j0.4
5	4 - 2	0.1 + j0.4

Line charging admittances may be neglected (JULY/AUGUST 2005 - 10M)

Sol: from the given data PS network can be drawn as shown



Self and mutual admittances are

$$Y_{11} = \frac{1}{0.1+j0.4} + \frac{1}{0.08+j0.4} = 1.069 - j4.756$$

$$Y_{22} = \frac{1}{0.1+j0.4} + \frac{1}{0.15+j0.75} + \frac{1}{0.1+j0.4} = 1.433 - j5.988$$

$$Y_{33} = \frac{1}{0.15+j0.75} + \frac{1}{0.2+j0.8} = 0.55 - j2.458$$

$$Y_{44} = \frac{1}{0.08+j0.4} + \frac{1}{0.1+j0.4} + \frac{1}{0.2+j0.8} = 1.363 - j5.933$$

$$Y_{12} = Y_{21} = -\left(\frac{1}{0.1+j0.4}\right) = -0.588 + j2.353$$

$$Y_{13} = Y_{31} = 0$$

$$Y_{14} = Y_{41} = -\left(\frac{1}{0.08+j0.4}\right) = -0.48 + j2.404$$

$$Y_{23} = Y_{32} = -\left(\frac{1}{0.15+j0.75}\right) = -0.256 + j1.282$$

$$Y_{24} = Y_{42} = -\left(\frac{1}{0.1+j0.4}\right) = -0.588 + j2.353$$

$$Y_{34} = Y_{43} = -\left(\frac{1}{0.2+j0.8}\right) = -0.294 + j1.176$$

$$\therefore Y_{BUS} = \begin{bmatrix} 1.069-j4.756 & -0.588+j2.353 & 0 & -0.48+j2.404 \\ -0.588+j2.353 & 1.433-j5.988 & -0.252+j1.282 & -0.588+j2.353 \\ 0 & -j0.256+j1.282 & 0.55-j2.458 & -0.294+j1.176 \\ -0.48+j2.404 & -0.588+j2.353 & -0.294+j1.176 & 1.363-j5.933 \end{bmatrix}$$